Pricing Buy-ins and Buy-outs

By

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Abstract

Pension buy-ins and buy-outs have become an important aspect of managing pension risk in recent years. As a step toward understanding these pension de-risking instruments, we develop models for pricing investment risk and longevity risk embedded in pension buy-ins and buy-outs. We also bring a contingent-claims framework to price credit risk of buy-in bulk annuities. Overall, our model can be used to assess the pricing of investment, longevity, and credit risks being transferred in pension buy-in and buy-out transactions.

Keywords: defined benefit pension plan, pricing, buy-ins, buy-outs.

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1. Introduction

The phrase “pension de-risking” has recently become part of the pension plan lexicon. According to a survey by the Association of Consulting Actuaries, 40% of UK employers with defined benefit (DB) plans indicated that they hoped to buy-in or buy-out all of their pension liabilities in the next ten years (Association of Consulting Actuaries, 2012). Such a trend of pension de-risking is driven by pension deficits due to the latest market downturns, low interest rate environments, new pension accounting standards, and prolonged life expectancy of retirees.

The 2007-2009 credit crisis and the subsequent drop in discount rates caused serious funding deficits of many DB pension plans. Despite corporate contributions and a rebound in equities afterwards, pension deficits persist as is evident by the fact that the plans of FTSE 100 companies were only 91% funded in 2013 (Lane Clark & Peacock LLP, 2013). In reaction to this, more stringent accounting standards set out new approaches to reflect changing circumstances and to promote transparency. For example, effective on January 1, 2013, the revised international pensions standard IAS19 requires that interest on pension scheme assets be calculated using a discount rate based on corporate bonds, rather than an expected return based on the actual assets held. Due to this, using 2012 data to illustrate possible effects from this change, Lane Clark & Peacock LLP (2013) shows that total 2012 profits of FTSE 100 companies will be £2 billion lower when recalculated under the new version of IAS19. In addition to market and regulatory risks, longevity risk has recently emerged as another significant concern for DB plans. Unanticipated mortality improvements increase the value of pension liabilities through longer lifetime annuity payments.

As pension liabilities become increasingly visible on the balance sheet following more stringent accounting standards, fluctuations in financial markets and increases in life expectancy can have a major and immediate impact on a firm’s share price. Growing pension deficits reduce internal financial resources and make the firm riskier and more difficult to access credit. As a result, the firm may have to forgo otherwise profitable projects. Rauh (2006) finds that mandatory pension contributions reduce a firm’s investment in desired projects. Campbell et al. (2012) document a positive relationship between mandatory pension contributions and the cost of capital. Their results imply that mandatory pension contributions increase a firm’s borrowing cost and limit a firm’s ability to access credit. Consistent with this, Lin et al. (2015) find a positive relation between pension exposures and cost of debt. In response
to those challenges, capital markets have developed a few solutions for DB plans to off-load risks such as pension buy-ins and pension buy-outs (Lin et al., 2014). Indeed, 2013 had a strong start with buy-in and buy-out deals worth more than £5.5 billion, including the record buy-out of £1.5 billion by EMI with Pension Insurance Corporation (Hawthorne, 2013).

There exists a rich literature that explores pension buy-in and buy-out activities (Deutsche Bank, 2011; LCP, 2012; Coughlan et al., 2013; LCP, 2014; Geddes et al., 2014). Both buy-ins and buy-outs involve pension plans buying bulk annuities from insurance companies. A market for buy-ins and buy-outs will develop if they are effective, economically affordable, and transparently priced. A difference between pension buy-outs and pension buy-ins arises from credit risk. Pension buy-outs are the most direct way to take pension liabilities off balance sheets. Thus, firms sponsoring DB plans with buy-outs are not subject to counter-party risk. However, a buy-in bulk annuity is written in the name of the pension trustee and the liabilities remain in the firm. As obligations of buy-in insurers are usually not fully collateralized and guaranteed by third parties, credit risk arises (Roy, 2012). Jerry Gandhi, the Group Pensions Director at RSA Insurance Group, highlighted the impact of credit risk on firms with buy-ins: “Our largest concern was the risk of the counter-party defaulting and this of course was a concern for the trustees and the company. The trustees in their own right would be able to get rid of the risk but in event of default, it would come back to the scheme, which would then sit back with the company” (Deutsche Bank, 2011).

When a practical means to entirely remove credit risk does not exist, we need a promising model in which to quantify buy-in counter-party risk. Indeed, much of the prior research ignores the cost of credit risk in buy-in transactions and applies the same pricing model for buy-ins and buy-outs. To fill the gap, following Cummins (1988) and Phillips et al. (1998), this paper brings a contingent-claims framework to the pricing of pension buy-in bulk annuities. With this setup, we are able to calculate buy-in credit risk premiums. In practice, there may exist factors (e.g., capital and reinsurance) that act

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1There are other differences between a buy-in and a buy-out: A pension plan sponsor can pay recurring premiums for a buy-in deal over a period of time, whereas it has to pay a single lump sum premium to purchase a buy-out. With buy-ins, pension participants do not see any change in the administration of their plans (Procter, 2014). In contrast, buy-outs transfer the administration and associated costs to insurers. In this paper, we do not explicitly model these differences between a buy-out and a buy-in as they are more firm or deal specific. In reality, pension plan sponsors can readily extend our model by incorporating these differences to fit their specific needs when they try to decide upon their buy-in or buy-out insurance providers.
to mitigate an insurer’s insolvency risk. As such, our calculated credit risk premium can be viewed as an upper bound for a scheme that purchases buy-in annuities.

While our model implies that buy-ins demand lower premiums due to credit risk, the prices of both buy-ins and buy-outs should be determined by the magnitude of investment risk and longevity risk transferred to insurers. Investment risk premium is a form of compensation for an insurer who bears the risk of uncertain pension asset returns. When pension assets fall below pension liabilities, capital infusion by the insurer is required to back its promises to pay the plan’s retirees or the trustee. Thus, we can view such promises as analogous to funding guarantee options sold by the insurer to the pension plan sponsor. In addition to investment risk, longevity risk is another major risk shifted to an insurer in a buy-in or buy-out deal. Longevity risk arises from unexpected gains in life expectancy (Cox et al., 2013). It is significant to a DB pension scheme when measured from a financial perspective. According to the International Monetary Fund (2012), each additional year of life expectancy increases the present value of liabilities of a typical DB plan by about 3-4 percent. We notice there are only a few preliminary papers on pricing these two risks for bulk annuities. Developing asset pricing theory in this area is important since it will help market participants better understand these pension risk management instruments. To extend the existing literature, as the second objective of this paper, we show how to explicitly price investment risk premium and longevity risk premium.

The paper is organized as follows. Section 2 presents the basic framework for a DB pension plan and introduces a model to price the investment premium and longevity risk premium of a bulk annuity. We also propose a model to price the insolvency put embedded in a buy-in. Section 3 compares the buy-in and buy-out prices subject to different parameter assumptions. Then we provide a numerical example to illustrate how to implement our model to price buy-ins and buy-outs in Section 4. The last section concludes.

2. Modeling Approach

2.1. Nature of the Liabilities. Consider a DB pension plan with a retired cohort of \( N(0) \) members at age \( x_0 \) at time 0. We assume that all retirees of the pension plan receive the same pension payment, and the plan is sufficiently large, so the idiosyncratic timings of individual deaths can be ignored. Let \( P \) denote the plan’s promised annual payment to each retiree who survives at the end of each year. In the
UK, the majority of pension benefits are linked to prices. To capture this effect in our model, \( P \) is in real value, obtained by removing the effects of general price-level changes over time.\(^2\) Without de-risking, the pension liability at time \( t \), \( PL_t \), is defined as the present value of future benefit obligations,

\[
PL_t = N(t) \cdot Pa_{x_0+t} \quad t = 1, 2, \ldots,
\]

where \( N(t) \) is the number of survivors at time \( t \), and \( a_{x_0+t} \) is the conditional expected value of immediate life annuity for age \( x = x_0 + t \) equal to

\[
a_x = a_{x_0+t} = \sum_{s=1}^{\infty} v_p s \bar{p}_{x,t}.
\]

In (2), the one-year discount factor \( v_p = 1/(1 + r_p) \) is calculated from the real discount rate \( r_p \) that determines the reserves for this annuity.\(^3\) The conditional expected \( s \)-year survival rate for age \( x \) at time \( t \), \( s\bar{p}_{x,t} \), equals:

\[
s\bar{p}_{x,t} = E \left[ s\bar{p}_{x,t} | \bar{p}_{x,t}, \bar{p}_{x+1,t+1}, \ldots, \bar{p}_{x+s-1,t+s-1} \right],
\]

where \( \bar{p}_{x+k-1,t+k-1}, k = 1, 2, \ldots \), is the probability that a plan member at age \( x + k - 1 \) at time \( t + k - 1 \) survives to age \( x + k \) at the beginning of year \( t + k \) (and gets a benefit payment) given the mortality table at time \( t + k - 1 \).

2.2. Longevity Risk Model.

2.2.1. The Lee and Carter (1992)’s Mortality Model. In this paper, we apply the Lee and Carter (1992) model to describe mortality dynamics:

\[
\log m_{x,t} = \delta_x + b_x \gamma_t + \epsilon_{x,t},
\]

where \( \log m_{x,t} \) is the logarithm of the central death rate \( m_{x,t} \) for age \( x \) \((x = 0, 1, 2, \ldots)\) in year \( t \) \((t = 1, 2, \ldots, K)\), and the constants \( \delta_x \) and \( b_x \) are the age-specific parameters. The normally distributed error term \( \epsilon_{x,t} \) has a zero mean that captures the age-specific transitory shock. The time-series common risk factor \( \gamma_t \) affects the mortality rates of all ages in year \( t \). In Lee and Carter (1992), it is modeled as

\(^2\)In practice, there will be a range of ages of the retirees in a plan instead of a single age \( x_0 \) as in our setup. Moreover, the retirees may all have different benefit payments, not a single one \( P \) as shown here. Our model can readily be extended to these more complex situations.

\(^3\)When \( P \) is in real value, the real discount rate should be used to incorporate the inflation risk.
a random walk with a drift $g$,

$$\gamma_t = \gamma_{t-1} + g + e_t, \quad e_t \sim N(0, \sigma_\gamma),$$  \hspace{1cm} (5)$$

where the error term $e_t$ is independent and identically distributed.

We assume the plan cohort has the same mortality experience as the UK male population. To estimate model (4), we use the UK male population mortality tables for ages 65 to 104 from 1950 to 2013 in the Human Mortality Database.\footnote{Available at \url{www.mortality.org} (data downloaded on September 1, 2015).} Lee and Carter (1992) propose a two-stage estimation process to estimate the Lee-Carter model (4). This estimation approach has been largely superseded by more modern fitting techniques, because it does not take into account the heterogeneity of deaths at different ages. Brouhns et al. (2002), for instance, use the maximum likelihood method to estimate the Lee-Carter model. Brouhns et al. (2002)’s method takes into account this heterogeneity and gives a better fit to the available data. Specifically, their approach assumes that the number of deaths for each age and period follows a Poisson distribution:

$$D_{x,t} \sim \text{Poisson}(m_{x,t} E_{x,t}),$$  \hspace{1cm} (6)$$

where $D_{x,t}$ is the number of deaths at age $x$ during year $t$, from an initial exposure-to-risk $E_{x,t}$. Following the maximum likelihood estimation procedure in Brouhns et al. (2002), we estimate the parameters in the Lee-Carter model (4). Figure 1 shows the estimated $\gamma_t$’s, which suggest $g = -0.41$ and $\sigma_\gamma = 0.85$ for (5).\footnote{To conserve space, the parameter estimates of $\delta_x$ and $b_x$ are not reported but available upon request.}

Then we use the Lee-Carter model to predict future mortality rates and generate $N(t)$, the random number of survivors at time $t$. Specifically, we assume all of the retirees in the plan are at the age $x_0$ at time $t = 0$. To forecast central death rates $\tilde{m}_{x_0+t,t}$ for age $x_0 + t$ at $t = 1, 2, \ldots$, we first simulate $\tilde{\gamma}_t$ where $t = 1, 2, \ldots$. With the estimated $\delta_{x_0+t}$’s and $b_{x_0+t}$’s as well as the simulated $\tilde{\gamma}_t$’s, we predict future mortality rates based on model (4) as follows:

$$\tilde{m}_{x_0+t,t} = \exp(\delta_{x_0+t} + b_{x_0+t}\tilde{\gamma}_t), \quad t = 1, 2, \ldots.$$
To simplify notation, now we use $\tilde{m}_{x,t}$ in place of $\tilde{m}_{x_0+t,t}$ where $x = x_0 + t$. The predicted one-year death rates $\tilde{q}_{x,t}$ can be calculated from $\tilde{m}_{x,t}$ using the following formula:

$$
\tilde{q}_{x,t} = \frac{2 \times \tilde{m}_{x,t}}{2 + \tilde{m}_{x,t}}.
$$

(7)

Then, the forecasted one-year survival probability equals

$$
\tilde{p}_{x,t} = 1 - \tilde{q}_{x,t}.
$$

2.2.2. **Modeling the Longevity Risk Premium.** The most popular mortality pricing methods can be divided into three categories: the Wang transform, the Sharpe ratio rule, and the risk-neutral method (Chen et al., 2010; Li and Ng, 2011). Chen et al. (2010) study the robustness of these three methods and conclude that for long maturities, compared to the other two methods, the Wang transform is a preferable one with respect to the perturbations of risk loadings and underlying parameters. As buy-in and buy-out annuities are long-term products, we use the Wang transform to price longevity risk throughout this work.

Wang (1996, 2000, 2001, 2002) develops a method of pricing insurance risks by transforming the underlying distribution. The transform has a clear economic interpretation because it can recover the capital asset pricing model (CAPM) and the Black-Scholes option formula. To apply it in the buy-out
Pricing, we can first use the observed prices of pure longevity securities to determine the parameter of the Wang transform and then use it to derive the longevity risk premium of a buy-out contract.

Consider a cumulative distribution function (cdf) $F(x)$ for a random payment $X$ paid at time $T$. The Wang transform “distorts” $F(x)$ to obtain a transformed distribution $F^*(x)$ according to the following equation:

$$F^*_X(x) = \Phi[\Phi^{-1}(F_X(x)) - \lambda],$$

(8)

where $\Phi(x)$ is the standard normal cdf and $\lambda$ is the market price of risk. Then, the fair price of $X$ equals the discounted expected value using the transformed distribution $F^*_X(x)$.

In the context of longevity risk transfer, according to Lin and Cox (2005), the Wang transform in (8) can be explicitly written as

$$F^*_T(x,0)(t) = \Phi[\Phi^{-1}(F_T(x,0)(t)) - \lambda],$$

(9)

where $T(x,0)$ is the lifetime of a person at age $x$ in time 0. Equation (9) can also be written as

$$s\bar{q}^*_x,0 = \Phi[\Phi^{-1}(s\bar{q}_x,0) - \lambda],$$

(10)

where $s\bar{q}_x,0$ is the expected probability that a person aged $x$ at time 0 dies before age $x + s$ and $s = 1, 2, \cdots$:

$$s\bar{q}_x,0 = 1 - s\bar{p}_x,0 = E[s\tilde{p}_x,0 | \tilde{p}_x,0, \tilde{p}_{x+1,0}, \cdots, \tilde{p}_{x+s-1,0}].$$

(11)

In bulk annuities, the market price of risk $\lambda > 0$ in (10) reflects the level of systematic longevity risk and firm–specific unhedgeable longevity risk assumed by the insurer (Cox and Lin, 2007; Lin and Cox, 2008). We assume the market price of longevity risk, $\lambda$, is constant with time. Then, the transformed $s$-year survival rate for age $x$ at time 0, $s\tilde{p}^*_x,0$, is simply

$$s\tilde{p}^*_x,0 = 1 - s\bar{q}^*_x,0 = 1 - \Phi[\Phi^{-1}(s\bar{q}_x,0) - \lambda].$$

(12)
We can determine $\lambda$ from a longevity security with equation (12) so that at time 0 the price of the longevity security is the discounted expected value under the transformed probability $s\bar{p}^*_x,0$:

$$
\sum_{s=1}^{S} v_s \cdot s\bar{p}^*_x,0 = \sum_{s=1}^{S} v_s \cdot \left( 1 - \Phi[\Phi^{-1}(s\bar{q}_x,0) - \lambda] \right),
$$

where $v_s, s = 1, 2, \cdots, S$, is the $s$-year discount factor based on the risk-free rate.

Suppose the pension plan sponsor transfers its longevity risk to an insurer by purchasing a buy-out bulk annuity. We can derive the market price of risk $\lambda$ based on the longevity-risk security market, and then use the same $\lambda$ to price the longevity risk premium of the buy-out. To calculate the longevity risk premium for a buy-out bulk annuity, we can first calculate the immediate life annuity $a_x$ for age $x$ from a mortality table, and compare it with $a^*_x$ based on the transformed $s$-year survival probabilities $s\bar{p}^*_x,0, s = 1, 2, \cdots$. Then, the longevity risk premium of a buy-out bulk annuity equals:

$$
P_{\text{longevity}} = \frac{a^*_x}{a_x} - 1.
$$

2.3. Investment Risk Model.

2.3.1. Financial Market Model. We assume the funds of a UK bulk annuity insurer are invested in three assets: the S&P UK stock total return index $A_{1t}$, the Merrill Lynch UK Sterling corporate bond total return index $A_{2t}$, and the 3-month UK cash total return index $A_{3t}$. We describe the process of these three assets at time $t$, $A_t = (A_{1t}, A_{2t}, A_{3t})$, as a geometric Brownian motion:

$$
d \log A_t = (\alpha - \frac{1}{2}\sigma \circ \sigma)dt + \sigma \circ dW_t,
$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is the drift vector of the three assets; the vector $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ includes their instantaneous volatilities. $W_t = (W_{1t}, W_{2t}, W_{3t})$ is a standard Brownian motion vector with each element having a mean of 0 and a variance of $t$. The notation ‘$\circ$’ represents the Hadamard (entrywise) product of vectors.

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6While we model the asset prices as continuous time processes, a discretization is used for simulation of investment returns over annual periods in the later numerical illustrations.

7While a jump-diffusion model is often viewed as a better model to capture extreme events, a log-normal model has its great advantage in tractability and easiness in calibrating parameters. Since the investment weight of an annuity insurer in stocks is typically low (e.g., only around 10% on average among European insurers (FitchRatings, 2011)), its impact on the investment risk premium tends to be limited. Moreover, our test rejects the model with a jump for the Merrill Lynch UK Sterling corporate bond index process. Thus, here we use the correlated log-normal models without jumps to describe the dynamics of all three financial assets.
We further assume the three assets are correlated with a covariance matrix equal to

\[
\text{Cov}(W_t, W_t) = \Sigma t,
\]

where

\[
\Sigma = \begin{pmatrix}
\sigma_1^2 & \rho_{12} \sigma_1 \sigma_2 & \rho_{13} \sigma_1 \sigma_3 \\
\rho_{12} \sigma_1 \sigma_2 & \sigma_2^2 & \rho_{23} \sigma_2 \sigma_3 \\
\rho_{13} \sigma_1 \sigma_3 & \rho_{23} \sigma_2 \sigma_3 & \sigma_3^2
\end{pmatrix}
\]

and \(\rho_{ij}\) is the correlation coefficient between asset \(i\) and asset \(j\).

To account for the effect of inflation risk in a UK pension bulk annuity, we use real returns to model pension assets dynamics. Given this, inflation risk is a part of investment risk. We estimate the parameters of the S&P UK stock total return index, the Merrill Lynch UK corporate bond total return index, and 3-month UK cash total return index based on the monthly data from January 2006 to December 2013 from Datastream. Then we convert the estimated monthly parameters to the annual parameters. The UK consumer price index (CPI) annual rates from 2006 to 2013 are obtained from the Office for National Statistics in the UK. Our drift and volatility annual estimates of real returns are shown in Table 1 and the estimated correlation coefficients are as follows:

\[
\Sigma_{WW} = \begin{pmatrix}
1 & \rho_{12} & \rho_{13} \\
\rho_{12} & 1 & \rho_{23} \\
\rho_{13} & \rho_{23} & 1
\end{pmatrix} = \begin{pmatrix}
1 & 0.3483 & -0.1002 \\
0.3483 & 1 & -0.1772 \\
-0.1002 & -0.1772 & 1
\end{pmatrix}.
\]

Table 1 indicates that the S&P UK stock index has a higher expected annual real log return \((\alpha_1 = 0.0448)\) than the Merrill Lynch UK corporate bond index \((\alpha_2 = 0.0215)\) and the 3-month UK cash index \((\alpha_3 = 0.0001)\). In addition, the correlation matrix (18) shows that the stock index and the corporate bond index are positively correlated with a correlation coefficient of \(\rho_{12} = 0.3483\), while both are negatively correlated with the 3-month UK cash index.
2.3.2. Modeling the Investment Risk Premium. The valuation of the investment risk, an important component in pricing buy-out bulk annuities, can be viewed as the valuation of a funding guarantee option. As the pension liabilities are evaluated annually by regulators (as long as there are survivors in the retired cohort), this funding guarantee option is equivalent to a series of one-year put options on the pension plan, where the strike prices are the values of the pension liabilities at different valuation dates. Similar option-pricing frameworks have been widely used to value the embedded benefit guarantees in pension plans (Blake, 1998) and those in variable annuities (Milevsky and Posner, 2001; Milevsky and Salisbury, 2006; Bauer et al., 2008; Bacinello et al., 2011). Different from a typical pension plan, a bulk annuity contract is similar to a “closed” pension and the contract liabilities need to be evaluated based on the actual survivors of a pension cohort. To price the funding guarantee option, in this paper, we propose an explicit approach that is flexible and easy to use by dynamically tracing the bulk annuity liabilities.

Suppose the annual survival payments from the insurer to the retirees will be made at the end of each year. Let \( PA_t \) denote the value of pension assets at time \( t \). Suppose that the pension is fully funded at inception with \( PA_0 = PL_0 \) and the pension valuation occurs at the end of each year. At the end of each year, the pension assets are reduced by the promised annuity (or survival) payments; moreover, in the event of pension shortfalls, the bulk annuity insurer has obligations to inject cash to make up the deficits. Let \( PA_{t+} \) be the value of the pension assets after the annuity payments and supplementary contributions (if there are any) for \( t = 1, 2, \cdots \),

\[
PA_{t+} = \max \{ PA_t - N(t) \cdot P, PL_t \},
\]

where \( PL_t = N(t) \cdot Pa_x \), as defined in (1), is the value of the pension liabilities at time \( t \).

By investing in the aforementioned three assets, the value of the pension assets between annuity payment dates satisfies the following stochastic process:

\[
d \log PA_t = \left( \pi(t) \bullet \alpha - \frac{1}{2} \pi(t) \Sigma \pi(t)^\top \right) dt + (\pi(t) \circ \sigma) \bullet dW_t,
\]

where \( \pi(t) = (\pi_1(t), \pi_2(t), \pi_3(t)) \) are the weights of the pension portfolio invested in the S&P UK stock total return index \( (\pi_1(t)) \), the Merrill Lynch UK corporate bond total return index \( (\pi_2(t)) \), and the 3-month UK cash total return index \( (\pi_3(t)) \) at time \( t \) (see, e.g., Fernholz (2002)). Here the notation ‘\( \bullet \)’
denotes the inner product of vectors. Under the risk-neutral measure $Q$, the synthetic pension assets process (20) becomes

$$
\frac{d \log P_A_t}{dt} = \left( r - \frac{1}{2} \pi(t) \Sigma \pi(t)^\top \right) dt + (\pi(t) \circ \sigma) \cdot dW_t,
$$

where $r$ is the risk-free rate. In what follows, we assume that the pension portfolio is continuously rebalanced such that the weights $\pi(t) = \pi = (\pi_1, \pi_2, \pi_3)$ are constant throughout the buy-out contract period. In this case, $\sigma_W^2 = \pi \Sigma \pi^\top$ is also a constant and the synthetic pension portfolio follows a geometric Brownian motion:

$$
P_A_t = P_A_0 \exp \left( \left( r - \frac{1}{2} \sigma_W^2 \right) t + (\pi \circ \sigma) \cdot W_t \right).
$$

In addition, we assume that the investment risk is independent of the longevity risk. Given the number of survivors $N(t)$ in each year ($t = 1, 2, \cdots$), the risk-neutral price of the funding guarantee option of the buy-outs, $PV_{invest}(N(\cdot))$, equals

$$
PV_{invest}(N(\cdot)) = \sum_{t=1}^{\tau_N} v_t \cdot E^Q \left[ (PL_t + N(t) \cdot P - P_A_t, 0)^+ \right] - v_{\tau_N+1} \cdot E^Q \left[ P_A_{\tau_N+1} \right],
$$

where $\tau_N = \min \{ \lfloor t \rfloor : N(t) = 0 \}$ is the number of integer years that the last pensioner of the retired cohort can survive and $v_t$ ($t = 1, 2, \cdots$) is the $t$-year discount factor based on the risk-free rate $r$. The first component on the right-hand side of (23) is the present value of supplementary contributions whenever an underfunded event occurs, while the second component represents the released reserve after all the pensioners are deceased.

While all the investment calculations above are performed in the risk-neutral measure using the risk-free rate $r$, the present value of pension liabilities $PL_t$ in (23) is calculated using the valuation rate of interest $r_p$ for the annuity reserving purpose. In the UK, $r_p$ is close to the weighted market redemption yield on the invested assets (Daykin, 2001). Accordingly, asset allocation and the expected returns on assets held by an annuity insurer to fund future benefits determine $r_p$ and $PL_t$, and thus affect the investment risk premium.
The ultimate investment risk premium $P_{\text{invest}}$ can be obtained by taking the expectation on (23) divided by the initial pension liabilities $PL_0$.

$$P_{\text{invest}} = E[PV_{\text{invest}}(N(\cdot))]/PL_0.$$  \hspace{1cm} (24)

2.4. Credit Risk Model. In practice, several factors, such as capital and reinsurance, determine whether a firm is solvent on an ongoing basis. For example, Solvency II is a European Union Directive concerning the amount of capital and the solvency of insurance companies in all 28 Member States, including the UK. Under Solvency II, all insurance companies must meet a Minimum Capital Requirement (MCR). If an insurer fails to maintain its financial resources above the level of the MCR, it will be subject to an immediate supervisory action, possibly including closure of the business.\(^8\)

In this paper, we simplify our credit risk analysis by assuming that an insolvency occurs when an insurer’s total assets fall below total liabilities. This assumption can be easily relaxed by incorporating a MCR, that is, a minimum amount by which total assets should exceed total liabilities, to determine the solvency of a buy-in insurer. An option pricing approach is adopted to model a buy-in insurer’s default risk. The liabilities created by issuing buy-in bulk annuities are analogous to risky corporate debts. The buy-in insurer is assumed to issue a bulk annuity policy and receive a premium payment, similar to the proceeds of a bond issue. In return, it promises to make a survival payment to the plan at the end of each period contingent on no default. Using this bond analogy, the value of the buy-in policy equals a default risk-free loan in the amount of all promised payments minus a put option owned by the buy-in insurer. This put option is known as the insolvency put (Phillips et al., 1998). We consider this put option to the European style. In this section, we model and price a buy-in insolvency put.

2.4.1. Modeling the Insolvency Put of a Buy-in. To evaluate the insolvency put in a buy-in policy, we first analyze the default risk through the buy-in insurer’s total asset and liability processes. Let $A_t$ and $L_t$ be the values of the buy-in insurer’s total assets and liabilities from all business lines at time $t$ respectively. It is worth noting that the assets and liabilities considered in this section are the aggregate assets and liabilities of the buy-in insurer. They are different from the pension assets and liabilities considered previously in Section 2.3 on investment risk. At time 0, we define an initial wealth distribution coefficient $\alpha$ ($0 < \alpha < 1$) such that $L_0 = \alpha A_0$. Following Cummins (1988) and

\(^8\)Source: Lloyd’s Solvency II online tutorial, www.lloyds.com.
Phillips et al. (1998), the total asset process and the total liability process are given by

\[ dA_t = \mu_A A_t dt + \sigma_A A_t dW_{A,t}, \]  

and

\[ dL_t = \mu_L L_t dt + \sigma_L L_t dW_{L,t}, \]  

under the physical measure \( P \), where \( \mu_A (\mu_L) \) is the instantaneous growth rate of total assets (total liabilities), \( \sigma_A (\sigma_L) \) is the instantaneous total asset (total liability) volatility, and \( W_{A,t} (W_{L,t}) \) is the standard Brownian motion for the total assets (total liabilities). The total assets and the total liabilities are dependent on each other with a correlation coefficient \( \rho_{AL} \) (which satisfies \( dW_{A,t} \cdot dW_{L,t} = \rho_{AL} dt \)). Furthermore, the CAPM model is used to price the total asset and liability accounts, which implies that

\[ \mu_A = r + \theta_A, \]  

and

\[ \mu_L = r_L + \theta_L, \]  

where \( r \) is the risk-free rate and \( \theta_A \) and \( \theta_L \) are the market risk premia for holding the insurance total assets and liabilities. Following Cummins (1988), we call the term \( r_L \) the inflation rate of the total liabilities. The term \( r_L \) captures not only price inflation but also the occurrence of new claims as well as the payment of claims, suggesting that \( r_L \) could be higher or lower than the economy-wide inflation rate. Therefore, under the risk-neutral measure \( Q \), we have

\[ A_t = A_0 \exp \left\{ \left( r - \frac{\sigma_A^2}{2} \right) t + \sigma_A W_{A,t} \right\}, \]  

and

\[ L_t = L_0 \exp \left\{ \left( r_L - \frac{\sigma_L^2}{2} \right) t + \sigma_L W_{L,t} \right\}. \]  

In addition, we define the asset-liability ratio \( \xi_t \) as

\[ \xi_t \equiv \frac{A_t}{L_t} = \xi_0 \exp \left\{ \left( r - \frac{\sigma_A^2}{2} \right) - \left( r_L - \frac{\sigma_L^2}{2} \right) \right\} t + \left( \sigma_A W_{A,t} - \sigma_L W_{L,t} \right), \]  

where \( \xi_0 = 1/\alpha \) is the initial asset-liability ratio. A default event occurs when the buy-in insurer’s total assets are not sufficient to meet its total liability obligations. In other words, the default event occurs
at the first time the asset-liability ratio $\xi_t$ is less than 100%. Thus, the asset-liability ratio process is crucial in determining the time of default and the default amount.

In the UK, insurers are authorized and regulated by the Financial Conduct Authority (FCA) and Prudential Regulation Authority (PRA). The Financial Services Compensation Scheme (FSCS) will protect pension plans for a maximum of 100% of bulk annuity benefits in the event of default on or after July 3, 2015, while the protection before July 3, 2015 was only up to 90% of the benefits.\(^9\) Accordingly, the buy-in credit risk premiums are dependent on whether authority benefit protections exist. Suppose that all liability claims upon default have the same priority in the bankruptcy process. The default amount for the buy-in bulk annuity is then $100(1 - \xi_\tau)\%$ of the pension liabilities at time $\tau$, where $\tau = \min \{ t : \xi_t < 1, t = 1, 2, 3, \ldots \}$ is the observed default time.\(^10\) In other words, the discounted buy-in insolvency put payoff is $e^{-r\tau} PL_\tau (1 - \xi_\tau, 0)^+$. If the pension benefits are protected by an authority up to a limit $\varphi$ (e.g., in the UK, $\varphi = 0.9$ before July 2015), the discounted buy-in insolvency put payoff can be expressed as $e^{-r\tau} PL_\tau [(1 - \xi_\tau, 0)^+ - (\varphi - \xi_\tau, 0)^+]$. Therefore, the value of the buy-in insolvency put without an authority benefit protection, $Put_{credit}$, is given by

$$ Put_{credit} = E^Q \left[ e^{-r\tau} PL_\tau (1 - \xi_\tau, 0)^+ \right], $$

while the insolvency put with a recovery rate $\varphi$, $Put_{credit}(\varphi)$, is

$$ Put_{credit}(\varphi) = E^Q \left[ e^{-r\tau} PL_\tau ((1 - \xi_\tau, 0)^+ - (\varphi - \xi_\tau, 0)^+) \right]. $$

In what follows, we provide a closed-form pricing formula for the buy-in insolvency put, with and without authority benefit protections.

### 2.4.2. Pricing the Credit Risk Premium

We decompose the buy-in insolvency put into a series of one-year European put options. Each one-year option only covers the default event observed at the end of that year. The first one-year put will be exercised and provide a discounted payoff equal to $e^{-r} PL_1 (1 - \xi_1, 0)^+$ if the asset-liability ratio $\xi_1$ at the end of year 1 is less than 100%. Given no

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\(^10\)In reality, it might be difficult to monitor insolvency instantly. The default time may depend on the release of quarterly/yearly financial statements. Here we assume that the insurer’s default will only occur at the end of each year. The proposed model can also be applied to the quarterly case with minor changes.
default in prior years, the decomposed one-year put in year \( t \) \((t = 2, 3, \cdots)\) will be exercised if the asset-liability ratio \( \xi_t \) falls below 100% at the end of year \( t \), at which point all other one-year puts from year \( t + 1 \) will expire as worthless. In this case, the discounted payoff for such a put in year \( t \) is given by 
\[
e^{-rt}PL_t (1 - \xi_t, 0)^{+} \cdot 1 \left( \min_{0 \leq s \leq t-1} \xi_s \geq 1 \right).
\]
Clearly, to price these puts, we should first analyze the distribution of the asset-liability ratio at the beginning of year \( t \) with no prior excursion below 100%, which is stated in the following lemma:

**Lemma 2.1.** Let \( Y_t = \log \left( \frac{\xi_t}{\xi_0} \right) \) and \( m_Y^{t} = \min_{0 \leq s \leq t} Y_s \) be its running minimum. The defective density of \( Y_t \) with its running minimum \( m_Y^{t} \geq \log \alpha \) is given by

\[
Pr \left( m_Y^{t} \geq \log \alpha, Y_t \in dy \right) = \frac{1}{\sqrt{2\pi \sigma^2 t}} \left( \exp \left\{ -\left( -y + \mu t \right)^2/2\sigma^2 t \right\} - \exp \left\{ 2\mu \log \alpha - \left( -y + 2\log \alpha + \mu t \right)^2/2\sigma^2 t \right\} \right) dy,
\]

where \( \mu = \left( r - \frac{\sigma_A^2}{2} \right) - \left( r_L - \frac{\sigma_L^2}{2} \right) \) and \( \sigma^2 = \sigma_A^2 - 2\cdot\rho_{AL}\cdot\sigma_A\sigma_L + \sigma_L^2. \) Note that \( \sigma \) is the instantaneous volatility of the asset-liability ratio process.

The condition \( m_Y^{t} \geq \log \alpha \), which is equivalent to \( \min_{0 \leq s \leq t} \xi_s \geq 1 \), guarantees that default does not occur before time \( t \). Formula (34) can be obtained by the change of variable technique, from Proposition 3.2.1.1 in Jeanblanc et al. (2009). Given the density of the (log) asset-liability ratio shown in Lemma 2.1, the pricing formulas for the one-year insolvency puts are presented as follows:

**Proposition 2.2.** The price of a decomposed one-year buy-in insolvency put option without authority benefit protection can be evaluated through the following formulas:

(i) The value of the buy-in insolvency put covering the first year, \( \text{Put}_{\text{credit},1} \), is

\[
E^Q \left[ e^{-r}PL_1 (1 - \xi_1, 0)^{+} \right] = N \left( 0 \right) \cdot \bar{p}_{x_0,0} \cdot P_{x_0+1} \cdot e^{-r+\left( \mu + \frac{\sigma^2}{2} \right)} \cdot \text{Put} \left( \xi_0, 1, 1, \mu + \frac{\sigma^2}{2}, \sigma \right),
\]

where \( \text{Put} \left( S_0, K, T, r, \sigma \right) \) is the Black-Scholes price of a put option with current price \( S_0 \), strike price \( K \), time to maturity \( T \), risk-free rate \( r \) and volatility \( \sigma \). The term, \( e^{\mu + \frac{\sigma^2}{2}} \cdot \text{Put} \left( \xi_0, 1, 1, \mu + \frac{\sigma^2}{2}, \sigma \right) \), represents the first-year expected default deficit per dollar pension liabilities.

(ii) For any subsequent year \( t \) \((t = 2, 3, \cdots)\), conditioning on the asset-liability ratio at time \( t-1 \) and utilizing Lemma 2.1, the one-year put covering insolvency claim in year \( t \), with no default in
prior years, can be obtained as

\[
P_{\text{credit},t} = \mathbb{E}^{Q} \left[ e^{-rt} P_{L,t} (1 - \xi_t, 0)^+ \cdot 1 \left( \min_{0 \leq s \leq t-1} \xi_s \geq 1 \right) \right] \\
= N(0) \cdot t \bar{p}_{x_0,0} \cdot P_{\alpha_{x_0+t}} \cdot e^{-r(t-\frac{\sigma^2}{2})} \cdot P_{\text{put}^*}, \tag{36}
\]

where

\[
P_{\text{put}^*} = \int_{0}^{\infty} \text{Put} \left( \xi_0 e^y, 1, 1, \mu + \frac{\sigma^2}{2}, \sigma \right) \text{Pr} \left( m_{t-1}^Y \geq \log \alpha, Y_{t-1} \in dy \right).
\]

The term, \( e^{\mu + \frac{\sigma^2}{2}} \cdot P_{\text{put}^*} \), represents the expected default deficit of year \( t \) per dollar pension liabilities with no default prior to year \( t \).

Based on the pricing formulas in Proposition 2.2, the value of the embedded buy-in insolvency put \( P_{\text{credit}} \) is the total value of these one-year put options:

\[
P_{\text{credit}} = \sum_{t=1}^{\infty} P_{\text{credit},t}. \tag{37}
\]

Accordingly, we can calculate the buy-in credit risk premium \( P_{\text{credit}} \) as a percentage of the initial pension liabilities as follows:

\[
P_{\text{credit}} = \frac{P_{\text{credit}}}{P_L}. \tag{38}
\]

Comparing the pricing formulas (32) and (33), it is not difficult to see that the price of a buy-in insolvency put with authority benefit protections is equivalent to the price of a same buy-in insolvency put without protections minus the expected present value of the authority benefit protection option. The authority benefit protection option can be also evaluated through the formulas in Proposition 2.2 by changing the strike parameter \( K \) from 1 to \( \varphi \), i.e.

\[
P_{\text{authority}} = P_{\text{credit}} \big|_{K=\varphi}. \tag{39}
\]

3. PRICING BUY-OUTS AND BUY-INS

To protect itself from both short-term volatility and long-term uncertainty, the pension plan sponsor is considering to transfer all of its pension liabilities to an insurer and remove the risks associated with
its DB plan. It can purchase a pension buy-in or a pension buy-out contract. For simplicity, we assume both pension insurance polices require a lump sum payment up front made by the pension plan sponsor. In the following, we first study how to price a pension buy-out contract, and then we price a pension buy-in with a model that accounts for credit risk.

3.1. **Total Risk Premium of Buy-outs.** Suppose a pension plan sponsor implements the buy-out strategy and transfers the entire pension liabilities from its retired cohort to an insurer at a lump sum premium. In such a pension buy-out deal, the pension plan sponsor frees itself of any pension liabilities and assets and is no longer subject to pension investment risk and longevity risk. The insurer, the other party, takes on responsibility for those obligations and has to meet all existing pension regulatory requirements (GAO, 2009). If pension assets fall below pension liabilities, the insurer, as the new sponsor, is obligated to fill the funding gap. As a result, the insurer asks for compensation to cover its investment risk and longevity risk assumed in this transaction. In the previous section, we have shown how to price these two risk premiums. Thus, the total risk premium of a buy-out deal $P_{total,buyout}$ is the sum of the investment risk premium and the longevity risk premium:

$$P_{total,buyout} = P_{invest} + P_{longevity}. \quad (40)$$

3.2. **Total Risk Premium of Buy-ins.** Here we focus on how to price the cost of credit risk in a buy-in transaction. As with a pension buy-out, a pension buy-in occurs when an insurer takes over financial responsibility for covering benefits for a selection of pensioners. However, the pension liabilities remain in the balance sheet of the firm. This means that, if the buy-in insurer goes bankrupt, the firm will have to resume its responsibility and pay the future benefits to its retirees. The option that the buy-in insurer can default on the remaining pension payments can be viewed as a put option known as the insolvency put (Phillips et al., 1998). This implies that the price of the buy-in equals the price of the buy-out minus the value of the insolvency put.

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11DB plan participants view purchasing buy-in annuities positively because pension buy-ins not only insulate DB plans from investment risk and longevity risk but they also allow the participants to retain the Pension Protection Fund (PPF) protections in the UK (the Employee Retirement Security Act of 1974 (ERISA) and Pension Benefits Guaranty Corporation (PBGC) protections in the US) (US Department of Labor, 2013). When plan sponsors are engaged in buy-outs, however, the participants in the US are concerned about their financial security because with buy-outs they are not protected by the PBGC or ERISA, and the protections provided by state guaranty associations in most states are far below PBGC maximum coverage levels (National Retiree Legislative Network, 2013). On and after July 3, 2015, FSCS will cover up to 100% of bulk annuity benefits if a UK bulk annuity insurer is insolvent. As a result, the UK participants may view buy-outs more positively than the US participants.
The credit risk premium of a buy-in insolvency put with the guaranteed recovery rate \( \varphi \) is given by

\[
P_{\text{credit}}(\varphi) = \frac{P_{\text{ut credit}}(\varphi)}{PL_0} = \left( \frac{P_{\text{ut credit}} - P_{\text{ut credit}|K=\varphi}}{PL_0} \right).
\]

(41)

Similar to a buy-out insurer, a buy-in insurer bears both investment risk and longevity risk. Different from a buy-out buyer, a buy-in buyer is subject to the credit risk of its buy-in insurer. As a high default risk reduces the attractiveness of a buy-in deal, the risk premium of a buy-in annuity \( P_{\text{total, buyin}} \) is inversely related to the credit risk of the buy-in insurer. Accordingly, the risk premium of a buy-in annuity \( P_{\text{total, buyin}} \) is lower than that of a similar buy-out deal \( P_{\text{total, buyout}} \) by \( P^*_{\text{credit}} \):

\[
P_{\text{total, buyin}} = P_{\text{total, buyout}} - P^*_{\text{credit}} = P_{\text{invest}} + P_{\text{longevity}} - P^*_{\text{credit}},
\]

(42)

where

\[
P^*_{\text{credit}} = \begin{cases} 
P_{\text{credit}} & \text{without benefit protection} \\
\frac{P_{\text{credit}} - P_{\text{credit}}|K=\varphi}}{PL_0} & \text{with benefit protection}
\end{cases}.
\]

(43)

4. Calibration to Data and Results

In this section, we first use an example to illustrate how to price pension buy-ins and buy-outs in the UK bulk annuity markets, and we then conduct a sensitivity analysis to show the reliability of our pricing models. The publicly available financial information of the major players in the UK buy-in and buy-out markets, the UK population mortality tables, as well as the UK capital market data are used for our model calibrations and estimations. Finally, we show that our approach can also be adopted for the US buy-in and buy-out markets.

We set year 2013 as the base year. Suppose at the end of year 2013 or \( t = 0 \), all participants of a DB plan reach the retirement age \( x_0 = 65 \). This cohort has the same mortality experience as the UK male population. To transfer its pension risk, assume the pension plan sponsor purchases a bulk annuity from Company A, an active player in the UK pension buy-in and buy-out market. It had the total assets of £3.14 billion and the total liabilities of £2.74 billion at the end of 2013.

4.1. Longevity Risk Premium. In our example, we use the European Investment Bank (EIB) bond issued in November 2004 to determine the market price of risk for longevity risk. The EIB bond was the result of the co-operation among the BNP Paribas as the structurer/manager, the European
Investment Bank (EIB) as the issuer and the PartnerRe as the provider of analysis, expertise, and risk taking capacity. It provides a solution for financial institutions, such as pension plans, to hedge their long-term systematic longevity risks (Cairns et al., 2005). However, it did not sell. One of the major reasons is that it was an expensive way to transfer longevity risk (Lin and Cox, 2008). Thus, we can view the market price of risk derived from the EIB bond as an upper bound, and the longevity risk premium of a buy-in or buy-out contract should not exceed this level. The technique we will illustrate below, based on the EIB bond, can be similarly applied to other longevity securities if their complete trading information is available.

4.1.1. *The Structure of the 2004 EIB Bond.* The bond has a term of 25 years and a size of £540 million (775 million euros). The underlying population of the bond is the English and Welsh male population aged 65 years old. Their actual longevity experience is published annually by the Office for National Statistics. The bond’s cash flows equal a fixed annuity, £50 million, multiplied by the percentage of the reference population still alive at each anniversary. The cash flows, therefore, decline over time.

4.1.2. *Longevity Market Price of Risk of the EIB Bond.* Here we show how to estimate the market price of risk for longevity risk embedded in the EIB bond using the Wang transform:

\[
s^*_{\bar{q}x,0} = \Phi[\Phi^{-1}(s_{\bar{q}x,0}) - \lambda_{EIB}]. \tag{44}
\]

The interest rates are the gilt STRIPS on November 18, 2004. To calculate the expected values of \(s\)-year death rate \(s_{\bar{q}x,0}, t = 1, 2, \cdots, 25\), for the English and Welsh male population aged 65, we first use the maximum likelihood method in Brouhns et al. (2002) to re-estimate the Lee-Carter model based on the UK male population mortality tables for ages 65 to 104 from 1950 to 2003 in the Human Mortality Database\(^{12}\). We then use the calibrated Lee and Carter (1992) model to generate Monte Carlo simulations of projected mortality rates.

After transforming \(s_{\bar{q}x,0}\) and obtaining the distorted death rate \(s^*_{\bar{q}x,0}\) in (44), we can calculate the transformed \(s\)-year survival rate for age \(x\) at time 0, \(s^*_{\bar{p}x,0}\), as follows:

\[
s^*_{\bar{p}x,0} = 1 - s^*_{\bar{q}x,0} = 1 - \Phi[\Phi^{-1}(s_{\bar{q}x,0}) - \lambda_{EIB}]. \tag{45}
\]

\(^{12}\)Available at www.mortality.org (data downloaded on September 1, 2015).
The issuer of the EIB bond will hedge all the investment and longevity risks on this bond. As a result, the bond is rated AAA and the default risk is minimal. For simplicity, we ignore the default risk of the EIB bond. We solve the longevity market price of risk $\lambda_{EIB}$ for the English and Welsh male with the following equation:

$$5,400,000,000 = 50,000,000a^{*}_{65|25}$$

where $a^{*}_{65|25}$ is the present value of a 25-year immediate life annuity with £1 survival benefit per year, payable in installments at the end of each year. The survival rates in $a^{*}_{65|25}$ are the distorted rates based on (45). Our calculated longevity market price of risk for the EIB bond, $\lambda_{EIB}$, equals 0.0943. Next we will use it to calculate the longevity risk premium for the buy-in and buy-out.

4.1.3. **Longevity Risk Premium of Buy-ins and Buy-outs.** At time 0, the plan has a retired cohort aged $x_0 = 65$ that has the same mortality experience as the UK male population. Assuming no investment and credit risk, the price of a bulk annuity $a^*_{65}$ can be calculated based on the estimated $\lambda_{EIB} = 0.0943$:

$$a^*_{65} = \sum_{s=1}^{\infty} v^s s\ddot{p}^s_{65,0} = 12.2329,$$ (46)

where

$$s\ddot{p}^s_{65,0} = 1 - \Phi[\Phi^{-1}(s\bar{q}_{65,0}) - \lambda_{EIB}] = 1 - \Phi[\Phi^{-1}(s\bar{q}_{65,0}) - 0.0943].$$ (47)

Without transform, the price of a bulk annuity $a_{65}$ equals

$$a_{65} = \sum_{s=1}^{\infty} v^s \ddot{p}_{65,0} = 11.8607,$$ (48)

where $s\ddot{p}_{65,0}$ is the expected $s$-year survival rate of age 65 from simulations based on the Lee and Carter (1992) model forecasts without transform. This implies the longevity risk premium, $P_{\text{longevity}}$, of the bulk annuity equals:

$$P_{\text{longevity}} = \frac{a^*_{65}}{a_{65}} - 1 = 3.14\%.$$ (49)

4.2. **Investment Risk Premium.** We use the model proposed in Section 2.3.2 to price the investment risk premium of buy-ins and buy-outs required by Company A. In the UK, pension benefits are usually

13 According to Standard & Poor’s (2014), the average cumulative default rate over 7 years for AAA-rated issuers in Europe from 1981 to 2013 is 0%, while it is as high as 1.18% for BBB-rated European issuers in the same period. The longest time horizon of the average cumulative default rates reported in this rating study is 7 years for the European region.
adjusted to cost of living, implying that buy-ins and buy-outs also transfer inflation risk. To capture
the impact of inflation risk on pricing, we use real returns for pension assets and real values of pension
benefits $P$. That is, we include inflation risk as part of investment risk.

We perform the following analysis as if the transaction prices are quoted at the end of 2013 or at time
0. Assume Company A invests 10% of its assets in stocks, 85% in bonds, and 5% in cash or its equivalents. These weights are similar to the weights of a typical European insurer (FitchRatings, 2011). Given these asset weights and the parameter estimates of the three assets in Table 1, the investments held to back the pension liabilities have an expected real annual log-return of 2.27% and a volatility of 7.73%. In the UK, the discount rate used to calculate an annuity insurer’s reserves is determined by the expected yield on its invested assets. In the base scenario, we use the expected real return of Company A’s asset portfolio as the discount rate for its liabilities. That is, $r_p = 2.27\%$. We further assume that the nominal long-term annual risk-free rate is 4.5%. With an average inflation rate of 2.98% from 2006 to 2013,\(^{14}\) it implies a (continuous) real risk-free rate of $r = 1.47\%$.

Suppose that both the buy-in and buy-out deals have 10,000 pensioners with an annual survival
benefit of £60,000 per pensioner. Using the Lee and Carter (1992) model, we generate 5,000 scenarios
of the mortality rates. For each mortality scenario, due to the independence assumption between the
mortality and economic variables, 1,000 scenarios are simulated separately for the values of the pension
assets at different times using (22). Each sample path of the pension assets is generated based on a
discretization of 252 time steps (trading days) per year. After applying (23) and (24), we find that the
expected investment risk premium is 9.22% of the initial pension liabilities. That is, $P_{\text{invest}} = 9.22\%$.

4.3. Credit Risk Premium of Buy-ins. Company A expanded rapidly before 2010. For example, its
business accounted for 13% of total premiums written in the UK pension buy-in and buy-out market
in 2009. However, due to a highly competitive UK bulk annuity market in the recent years, Company
A failed to maintain its market share. Company A’s market share decreased to 6% and its profitability
went down in 2012. Standard & Poors Ratings Services lowered Company A’s credit rating from
‘A−’ (Strong) to ‘BBB+’ (Good) and placed its rating on CreditWatch negative in 2014. As a result,
the credit risk could be a concern to pension plans that will purchase or have purchased buy-in bulk
annuities from Company A.

\(^{14}\)Source: Office for National Statistics, UK.
In this section, the pricing formulas introduced in Section 2.4.2 are used to calculate the credit risk premium of buy-in bulk annuities issued by Company A. Company A is regulated by the PRA and FCA, which implies that 90% of Company A’s buy-in benefits are guaranteed by FSCS in 2014. To properly evaluate the credit risk embedded in Company A’s buy-ins, one should exclude the expected present value of the FSCS guarantee from Company A’s buy-in insolvency put. It can be calculated through the formulas in Proposition 2.2 with the same parameters, except the strike parameter being “$\varphi = 0.9$” instead of “1” (see Eq.(39)).

The proposed credit risk model in Proposition 2.2 depends on the following parameters: the real risk-free interest rate ($r$), the total liability inflation rate ($r_L$), the life annuity factors for age 65 and above ($a_x, x = 65, 66, \cdots$), the initial asset-liability ratio ($\xi_0$), the instantaneous volatilities of total assets and liabilities for Company A ($\sigma_A$ and $\sigma_L$) as well as their correlation coefficient ($\rho_{AL}$), and the recovery rate ($\varphi$). It is worth noting that the factors we do not include in the model (e.g., reinsurance and support from parent companies) can change the credit risk premiums reported here.

As noted earlier, the real risk-free rate is $r = 1.47\%$. Suppose the occurrence of new claims and the payment of claims offset each other. Then the liability inflation rate $r_L$ is assumed to be the average (continuous) UK inflation rate of 2.93% from 2006 to 2013. To estimate the immediate life annuity factors, $a_x, x = 65, 66, \cdots$, the real pension valuation rate of $r_p = 2.27\%$ is used to discount annuity cash flows. The 5,000 scenarios of mortality rates generated in the previous section are used again to evaluate the annuity factors $a_x, x = 65, 66, \cdots$.

The risk parameters of Company A’s total assets and liabilities are estimated using Company A’s annual financial data from 2008 to 2013. The total asset volatility of Company A equals $\sigma_A = 0.2571$ and its total liability volatility is $\sigma_L = 0.2860$. The correlation coefficient between Company A’s total assets and total liabilities $\rho_{AL}$ is estimated at 0.9851. Thus, the instantaneous volatility for the asset-liability ratio process is $\sigma = 0.0551$. We assume Company A’s year-end asset-liability ratio in 2013 is $\xi_0 = 1.10$. The FSCS guaranteed recovery rate in 2014 equals $\varphi = 0.9$. Based on the above parameter values, Proposition 2.2 and (41), the credit risk premium of Company A’s buy-in bulk annuities with benefit protection is estimated at $P_{\text{credit}}(\varphi) = 0.18\%$. While the magnitude of this credit risk premium is lower than those of the investment risk premium and the longevity risk premium, the credit risk

\[ \text{Source: Office for National Statistics, UK.} \]
premium \( P_{\text{credit}}(\varphi) = 0.18\% \) implies a non-negligible value amount. For instance, for each £1 billion pension liability risk transfer, this risk premium will reduce Company A’s buy-in bulk annuity premium by £1,800,000.\(^{16}\)

4.4. **Sensitivity Analysis.** We now assess the impact of the parameter values. In doing these analyses, we explore how the implications of parameter choice affect buy-in and buy-out risk premiums. In general, the analyses below have illustrated that our formulation leads to intuitive results including how the risk premiums reflect unexpected mortality improvements, financial conditions of buy-in insurers, pension investment changes, etc. These help demonstrate the reliability of our pricing models.\(^{17}\)

4.4.1. **The Impact of Longevity Risk.** An unexpected mortality improvement increases pension liabilities. The market price of risk estimated from the Wang transform reflects the market belief on the longevity risk and in turn determines the longevity risk premium. To test the robustness of our model, we consider the sensitivity of the longevity risk premium to the market price of risk \( \lambda \) that accounts for the level of systematic longevity risk and firm-specific unhedgeable longevity risk in bulk annuities. Our analysis shows that the longevity risk premium \( P_{\text{longevity}} \) increases with the market price of risk \( \lambda \). For example, when the market price of risk \( \lambda \) is 0.1886, doubled from the base case 0.0943, the longevity risk premium \( P_{\text{longevity}} \) of Company A goes up from 3.14% to 6.16%, a 96.3% rise. This result makes intuitive sense as a higher risk premium is associated with a higher risk.

4.4.2. **The Impact of Investment Risk.** In practice, insurers manage their annuity portfolio by allocating assets to generate cash flows equal to the expected annuity payments (Daykin, 2001). To achieve a good hedge against the expected liability cash flows, annuity insurers heavily invest in fixed-income securities. We find that the investment risk premium increases when more funds are invested in the stock index. When the plan invests the pension funds equally in the stock index and the corporate bond index, the investment risk premium \( P_{\text{invest}} \) is more than doubled—21.76% compared with 9.22% in the base case where only 10% of the pension funds are invested in the stock index. This higher investment

\(^{16}\)The credit risk premium of a buy-in annuity is expected to be lower than those of corporate bonds and credit default swaps. This is because, for example, in the UK, in the event of insolvency, the FSCS will protect pension plans up to 90% of the bulk annuity benefits before July 2015, but corporate bonds or credit default swaps are not guaranteed by the FSCS. The UK Company A was rated “BBB+” in 2014. The credit risk premium for a 10-year “BBB” corporate bond was around 1.70% in January 2014 (Johnson, 2014), which is higher than our calculated credit risk premium of the buy-in annuity sold by this UK insurer, 0.18%. This is consistent with our expectation.

\(^{17}\)To conserve space, we do not report the results of sensitivity analysis in the paper. They are available upon request.
risk premium is explained by the mismatch between pension asset and liability durations. It is also explained by the higher risk from more investment in equity.

4.4.3. *The Impact of Credit Risk.* Several parameters influence the default probability and amount of a buy-in insurer. Below, we investigate how the initial asset-liability ratio $\xi_0$, total liability inflation rate $r_L$, and instantaneous volatilities of the asset-liability ratio $\sigma$ affect the buy-in credit risk premium.

*Different Initial Asset-Liability Ratios $\xi_0$.** Given that all other parameters are the same as those in the base scenario, we examine how the credit risk premiums (as a percentage of the initial pension liabilities), $P_{\text{credit}}(\varphi)$, change with asset-liability ratios $\xi_0$ for Company A. Our results show that the credit risk premium $P_{\text{credit}}(\varphi)$ decreases with $\xi_0$. This can be explained by the fact that holding more economic capital improves an insurer’s ability to resist shocks, and thus exposes it to a lower insolvency risk. As a result, the credit risk premium goes down.

*Different Total Liability Inflation Rates $r_L$.** In addition to the initial asset-liability ratio, we also compare the prices of buy-ins subject to different liability inflation risk parameters. The sensitivity analysis indicates that the credit risk premiums increase with the liability inflation rate $r_L$. This is consistent with the intuition that a higher liability growth tends to increase firm risk and thus the default probability if all other risk parameters stay the same. Moreover, the increment of credit risk premium in response to the liability growth change is amplified as the initial asset-liability ratio $\xi_0$ increases. When $\xi_0$ goes up, the credit risk premium of Company A dramatically increases with the total liability growth rate $r_L$.

*Different Instantaneous Volatilities of the Asset-Liability Ratio $\sigma$.** To explore the impact of instantaneous volatilities of the asset-liability ratio $\sigma$ on the credit risk premiums, we analyze two more insurers in the bulk annuity market, Company B and Company C.

Company B is a London-based UK insurer with total assets of £13.50 billion and total liabilities of £12.55 billion as of December 2013. Since their first bulk annuity transaction in 2006, Company B has become one of the major bulk annuity providers in the UK. They acquired 34% of the market share and were the largest bulk annuity seller of the year in 2012. Company C is another major player in the UK pension bulk annuity market with good financial standing. As of December 2013, Company C had total assets of £363.16 billion with an “A” (Excellent) financial strength rating from A.M. Best.
Our results show that the values of the credit risk premium are quite different among Companies A, B and C. Due to a relatively low asset-liability correlation coefficient $\rho_{AL}$, Company B has the highest instantaneous volatility ($\sigma = 0.0836$) for its asset-liability ratio process. Consequently, the credit risk premiums of Company B are much higher compared to those of Company A and Company C. For example, the credit risk premium of Company B is 0.78% higher than Company A (which is 0.18%) when $\xi_0 = 1.1$ and $r_L = 0.0293$. On the other hand, due to a high asset-liability correlation coefficient, the asset-liability ratio of Company C has a very small volatility ($\sigma = 0.0035$), which yields negligible credit risk premiums for all the scenarios tested. In sum, our analysis reveals that the more volatile the asset-liability ratio process is, the more valuable the insolvency put will be. This finding highlights the importance of asset-liability management to insurance companies. To reduce the default risk, in addition to holding more capital, insurers may also need to dynamically match their assets with their insurance claims so that they are protected against potential adverse changes arising from either the asset or the liability side.

**Different Recovery Rates $\varphi$.** All three companies are protected by FSCS with a guaranteed recovery rate of $\varphi = 0.9$ before July 3, 2015. It is important to understand how regulatory benefit protection affects credit risk premium. To answer this question, next we analyze credit risk premiums of Company A and Company B at different recovery rates. Our results show that as the recovery rate $\varphi$ decreases from 0.95 to 0.80, the credit risk premiums of Company A and Company B increase steadily. However, when the recovery rate $\varphi$ is below 0.80, the credit risk premiums do not go up further. This can be explained by the fact that most default events are likely to end up with not “too bad” asset-liability ratios (e.g., 0.80 or above in the tested scenarios) under annual audits. The light tail of the default distribution based on our model leads to a low value of the regulatory benefit protection option.

4.5. **Pricing Buy-ins and Buy-outs in the US Markets.** While the above modeling is specific for UK companies, it could be adopted for a different country by modifying our setup to reflect the regulatory standards of the country where the pension scheme is assumed to be located. For example, there exists a few differences between the US and UK pension regulations. In the UK, buy-ins can be used to transfer pension risk arising from a subset of members of a pension scheme, whilst buy-outs must be conducted for all members. However, in the US, buy-outs can be executed for just a subset of members
in a scheme. Despite this, we can still use the same models for the UK markets to price the US buy-in and buy-out annuities.

Moreover, departing from the UK pension law required for registered pension plans, pension benefits in the US are typically not indexed for inflation. To price buy-ins and buy-outs in the US markets, we can still apply the above methodology but use the nominal returns of different pension assets and the nominal value of annual survival payment $P$ for pricing. Since the inflation risk is not transferred, with the nominal returns, we expect a lower investment risk premium required by US bulk annuity insurers than what we calculate for the UK insurers in this section.

The business of insurance in the US is primarily monitored and regulated at the state level. The US insurance companies are periodically examined by state regulators and are required to submit audited quarterly & annual financial statements (NAIC, 2010). In the case of bankruptcy, state life and health insurance guaranty associations will generally resume the liabilities of an insolvent insurer to its policyholders up to a certain limit. In most states, this protection for an individual life is capped at $300,000. In Section 2.4, we model the credit risk premium in terms of the recovery rate $\varphi$, which is a fraction of pension benefits protected in the event of default. To price the credit risk premium for a buy-in annuity in the US, we can first estimate the recovery rate by dividing the protection limit (e.g., $300,000) by the promised pension payoffs to obtain $\varphi$. Given that $\varphi$ is not greater than 1, we can apply the same formula (33) to approximate the credit risk premium for a buy-in annuity sold in the US.

5. Conclusion

We are learning every day how important risk transfer is in the management of DB pension plans. Pension buy-ins and buy-outs can be an important set of solutions from the insurance industry to these problems. A market for pension buy-ins and buy-outs will develop if their prices are attractive to potential buyers and sellers. Thus, in this paper, we focus on quantifying and pricing risks embedded in the pension buy-in and buy-out transactions.

The price of a buy-out contract depends on the investment risk and longevity risk shifted to an insurer. Pricing a buy-in contract is richer in that, in addition to the investment risk and longevity risk premiums, we have to consider the scenario in which a buy-in insurer does not have enough resources

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to cover its shortfalls and defaults on its obligations to a pension plan. We find only a few preliminary papers on this topic. Developing theories in this direction is important because it will extend traditional annuity market models, and it could be very useful in explaining these pension de-risking instruments to potential pension plan sponsors. Motivated by this, we propose option pricing models to evaluate the default risk and the investment risk. The buy-in default risk can be considered as an insolvency put written by the insurer, while the investment risk can be viewed as a series of one-year put options with strike prices equal to the values of the pension liabilities at different valuation dates. We also apply the Wang transform to calibrate the longevity risk premium for bulk annuities using a security traded in the longevity-risk security market. Our sensitivity analyses show the robustness of our models. Our findings suggest that, to reduce default risk and improve contracting terms with buy-in buyers, insurers should hold sufficient risk capital and carefully manage their asset portfolios to align with their insurance liabilities.

Our paper is based on the assumption that investment risk, longevity risk, and credit risk are independent. While zero correlation between investment risk and longevity risk is a common assumption in the pension and mortality literature, a study on whether and how investment risk, longevity risk, and credit risk in bulk annuities are interacted will certainly be fruitful. We leave this question for future research.

REFERENCES


APPENDICES OF PRICING BUY-INS AND BUY-OUTS

1. Maximum Likelihood Estimates of $\delta_x$ and $b_x$ in the Lee-Carter Model

Table 1. This table shows the age-specific parameters $\delta_x$ and $b_x$ in the Lee and Carter (1992) model estimated with the maximum likelihood method proposed by Brouhns et al. (2002).

<table>
<thead>
<tr>
<th>Age</th>
<th>$\delta_x$</th>
<th>$b_x$</th>
<th>Age</th>
<th>$\delta_x$</th>
<th>$b_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>-3.6193</td>
<td>0.0444</td>
<td>85</td>
<td>-1.7747</td>
<td>0.0253</td>
</tr>
<tr>
<td>66</td>
<td>-3.5393</td>
<td>0.0422</td>
<td>86</td>
<td>-1.6789</td>
<td>0.0246</td>
</tr>
<tr>
<td>67</td>
<td>-3.4291</td>
<td>0.0435</td>
<td>87</td>
<td>-1.5967</td>
<td>0.0233</td>
</tr>
<tr>
<td>68</td>
<td>-3.3392</td>
<td>0.0426</td>
<td>88</td>
<td>-1.5173</td>
<td>0.0217</td>
</tr>
<tr>
<td>69</td>
<td>-3.2419</td>
<td>0.0421</td>
<td>89</td>
<td>-1.4300</td>
<td>0.0202</td>
</tr>
<tr>
<td>70</td>
<td>-3.1543</td>
<td>0.0409</td>
<td>90</td>
<td>-1.3502</td>
<td>0.0183</td>
</tr>
<tr>
<td>71</td>
<td>-3.0695</td>
<td>0.0388</td>
<td>91</td>
<td>-1.2775</td>
<td>0.0163</td>
</tr>
<tr>
<td>72</td>
<td>-2.9565</td>
<td>0.0399</td>
<td>92</td>
<td>-1.1796</td>
<td>0.0158</td>
</tr>
<tr>
<td>73</td>
<td>-2.8649</td>
<td>0.0394</td>
<td>93</td>
<td>-1.1062</td>
<td>0.0134</td>
</tr>
<tr>
<td>74</td>
<td>-2.7694</td>
<td>0.0385</td>
<td>94</td>
<td>-1.0314</td>
<td>0.0139</td>
</tr>
<tr>
<td>75</td>
<td>-2.6852</td>
<td>0.0373</td>
<td>95</td>
<td>-0.9694</td>
<td>0.0114</td>
</tr>
<tr>
<td>76</td>
<td>-2.5884</td>
<td>0.0367</td>
<td>96</td>
<td>-0.9031</td>
<td>0.0110</td>
</tr>
<tr>
<td>77</td>
<td>-2.5082</td>
<td>0.0348</td>
<td>97</td>
<td>-0.8398</td>
<td>0.0081</td>
</tr>
<tr>
<td>78</td>
<td>-2.4127</td>
<td>0.0343</td>
<td>98</td>
<td>-0.7794</td>
<td>0.0080</td>
</tr>
<tr>
<td>79</td>
<td>-2.3184</td>
<td>0.0335</td>
<td>99</td>
<td>-0.7219</td>
<td>0.0074</td>
</tr>
<tr>
<td>80</td>
<td>-2.2307</td>
<td>0.0311</td>
<td>100</td>
<td>-0.6674</td>
<td>0.0062</td>
</tr>
<tr>
<td>81</td>
<td>-2.1498</td>
<td>0.0290</td>
<td>101</td>
<td>-0.6159</td>
<td>0.0053</td>
</tr>
<tr>
<td>82</td>
<td>-2.0433</td>
<td>0.0290</td>
<td>102</td>
<td>-0.5673</td>
<td>0.0039</td>
</tr>
<tr>
<td>83</td>
<td>-1.9540</td>
<td>0.0279</td>
<td>103</td>
<td>-0.5217</td>
<td>0.0075</td>
</tr>
<tr>
<td>84</td>
<td>-1.8560</td>
<td>0.0274</td>
<td>104</td>
<td>-0.4789</td>
<td>0.0050</td>
</tr>
</tbody>
</table>

2. Sensitivity Analysis

This appendix provides more detailed explanations for Section 4.4 on sensitivity analysis.

The Impact of Longevity Risk. To test the robustness of our model, we consider the sensitivity of longevity risk premium to the market price of risk $\lambda$ that accounts for the level of systematic longevity risk and firm-specific unhedgeable longevity risk in bulk annuities. Table 2 presents the longevity risk premium at different levels of $\lambda$ given other parameters are the same as those in Section 4.1.

Date: November 5, 2015.
TABLE 2. Longevity Risk Premiums $P_{\text{longevity}}$ with Different Market Prices of Risk $\lambda$

<table>
<thead>
<tr>
<th>Market Price of Risk ($\lambda$)</th>
<th>$P_{\text{longevity}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0472</td>
<td>1.58%</td>
</tr>
<tr>
<td>0.0943</td>
<td>3.14%</td>
</tr>
<tr>
<td>0.1415</td>
<td>4.67%</td>
</tr>
<tr>
<td>0.1886</td>
<td>6.16%</td>
</tr>
</tbody>
</table>

It is clear from the above table that as the market price of risk $\lambda$ increases in magnitude, the longevity risk premium $P_{\text{longevity}}$ becomes higher. This result makes intuitive sense as a high risk premium is associated with the presence of a high level of risk. Note that the systematic longevity risk and firm-specific unhedgeable longevity risk has a significant impact on the longevity risk premium. When the market price of risk $\lambda$ is 0.1886, doubled from the base case 0.0943, the longevity risk premium $P_{\text{longevity}}$ of Company A goes up from 3.14% to 6.16%, a 96.3% rise.

**The Impact of Investment Risk.** Table 3 shows how the investment risk premium of Company A changes with the investment weights $\pi = (\pi_1, \pi_2, \pi_3)$ in the S&P UK stock total return index ($\pi_1$), the Merrill Lynch UK corporate bond total return index ($\pi_2$), and the 3-month UK cash total return index ($\pi_3$). Other parameters remain the same as those in the base scenario. We find that the investment risk premium increases when more funds are invested in the stock index. When the plan invests the pension funds equally in the stock index and the corporate bond index, the investment risk premium $P_{\text{invest}}$ is more than doubled—21.76% compared with 9.22% in the base case.

**TABLE 3. Investment Risk Premiums of Company A’s Buy-out/Buy-in Bulk Annuities with Different Investment Weights $\pi = (\pi_1, \pi_2, \pi_3)$**

<table>
<thead>
<tr>
<th>Investment Weights ($\pi$)</th>
<th>$P_{\text{invest}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5%, 90%, 5%)</td>
<td>7.85%</td>
</tr>
<tr>
<td>(10%, 85%, 5%)</td>
<td>9.22%</td>
</tr>
<tr>
<td>(15%, 80%, 5%)</td>
<td>10.59%</td>
</tr>
<tr>
<td>(20%, 75%, 5%)</td>
<td>11.97%</td>
</tr>
<tr>
<td>(50%, 50%, 0%)</td>
<td>21.76%</td>
</tr>
</tbody>
</table>

**The Impact of Credit Risk.**

Different Initial Asset-Liability Ratios $\xi_0$. We first examine the sensitivity of the credit risk premiums to different initial asset-liability ratios. Historically, Company A’s asset-liability ratio at the beginning of each year varied from 1.097 to 1.384. We use this to set the range of $\xi_0$ for our sensitivity analysis.

Given all other parameters are the same as those in the base scenario, the fourth column of Table 4 shows the credit risk premiums (as a percentage of the initial pension liabilities), $P_{\text{credit}}(\phi)$, at different asset-liability ratios $\xi_0$ for Company A. The scale of the credit risk premiums lies from several basis points to 0.42% when $r_L$ equals 0.0293 and $\xi_0$ is between 1.05 and 1.30. Given the huge size of a typical buy-in deal, our results show that the credit risk premium is not negligible and should be an important component in the buy-in pricing framework. For instance, one basis...
point rise in the credit risk premium will increase the bulk annuity premium by £100,000 for a £1 billion buy-in deal. Note that the credit risk premium \( P_{\text{credit}}(\varphi) \) decreases significantly when \( \xi_0 \) jumps up to 1.4.

Different Total Liability Inflation Rates \( r_L \). In addition to the initial asset-liability ratio, we also compare the prices of buy-ins subject to different liability inflation risk parameters. The credit risk premiums of Company A with a higher total liability growth rate at \( r_L = 0.04 \) (approximately 1% increase) are presented in the fifth column of Table 4.

We can see from the table that the credit risk premiums increase significantly when the liability inflation rate \( r_L \) changes from 0.0293 to 0.04. This is consistent with the intuition that a higher liability growth tends to increase firm risk and thus the default probability if all other risk parameters stay the same. Moreover, the increment of credit risk premium in response to the liability growth change is amplified as the initial asset-liability ratio \( \xi_0 \) increases. When the total liability growth rate \( r_L \) goes up from 0.0293 to 0.04, the credit risk premium is only about a 50% increase in the case of \( \xi_0 = 1.05 \); but it dramatically increases by five times at \( \xi_0 = 1.30 \).

**Table 4. Credit Risk Premiums \( P_{\text{credit}}(\varphi) \) of Buy-in Bulk Annuities Issued by Company A, B and C**

<table>
<thead>
<tr>
<th>A/L Ratio</th>
<th>Comp. B (( \sigma = 0.0836 ))</th>
<th>Comp. A (( \sigma = 0.0551 ))</th>
<th>Comp. C (( \sigma = 0.0035 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_0 )</td>
<td>( r_L = 0.0293 )</td>
<td>( r_L = 0.04 )</td>
<td>( r_L = 0.0293 )</td>
</tr>
<tr>
<td>1.05</td>
<td>1.40%</td>
<td>1.73%</td>
<td>0.42%</td>
</tr>
<tr>
<td>1.10</td>
<td>0.96%</td>
<td>1.28%</td>
<td>0.18%</td>
</tr>
<tr>
<td>1.20</td>
<td>0.53%</td>
<td>0.82%</td>
<td>0.04%</td>
</tr>
<tr>
<td>1.30</td>
<td>0.31%</td>
<td>0.55%</td>
<td>0.01%</td>
</tr>
<tr>
<td>1.40</td>
<td>0.18%</td>
<td>0.37%</td>
<td>( 2.4 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

Different Instantaneous Volatilities of the Asset-Liability Ratio \( \sigma \). To explain the impact of insurers’ asset and liability management on the credit risk premium, we analyze two more insurers in the bulk annuity market, Company B and Company C.

The asset and liability volatilities of Company B are estimated at 0.1161 and 0.0888, respectively, with a correlation coefficient of 0.6969. Company B’s year-end asset-liability ratios were around 1.07–1.09 from 2010 to 2013. On the other hand, Company C had total asset and liability volatilities of \( \sigma_A = 0.1141 \) and \( \sigma_L = 0.1154 \) with a high correlation coefficient of \( \rho_{AL} = 0.9960 \). Company C’s asset-liability ratio varied between 1.014 and 1.047 from 1996 to 2013. Similar to Company A’s deals, the buy-in bulk annuities issued by Company B and Company C were also protected by FSCS with a guaranteed recovery rate of \( \varphi = 0.9 \) in 2014.

Given the above total asset and liability estimates, with all other parameters the same as those in the base case, Table 4 provides the estimated credit risk premiums of buy-in annuities of Company B and Company C at different asset-liability ratios \( \xi_0 \) and total liability inflation rates \( r_L \). They show a similar trend as that in Company A’s case. However, the values of the credit risk premium are quite different for these three companies. Due to a relatively low asset-liability correlation coefficient \( \rho_{AL} \), Company B has the highest instantaneous volatility \( (\sigma = 0.0836) \) for its asset-liability ratio process. Consequently, the credit risk premiums of Company B are much higher compared to those of Company A and Company C. For example, the credit risk premium of Company B is 0.78% higher than Company A (which is 0.18%) when \( \xi_0 = 1.1 \) and \( r_L = 0.0293 \). On the other
hand, due to a high asset-liability correlation coefficient, the asset-liability ratio of Company C has a very small volatility ($\sigma = 0.0035$), which yields negligible credit risk premiums for all the scenarios tested. Clearly, Table 4 reveals that the more volatile the asset-liability ratio process is, the more valuable the insolvency put will be.

**Different Recovery Rates $\phi$.** Next we analyze credit risk premiums of Company A and Company B at different recovery rates $\phi$. Table 5 shows the credit risk premiums of buy-in annuities of Company A and Company B at different recovery rates $\phi$ and initial asset-liability ratios $\xi_0$. The other parameters stay the same as those in the base scenario. For both companies, their credit risk premiums are lower at a higher regulatory guaranteed recovery rate $\phi$. As the recovery rate $\phi$ decreases from 0.95 to 0.8, the credit risk premium increases steadily, and it eventually rises to a stable level. It also implies that the guaranteed protection with a recovery rate of 0.80 (or lower) cannot reduce the credit risk premiums for the tested scenarios.

<table>
<thead>
<tr>
<th>Recovery Rate ($\phi$)</th>
<th>Comp. A ($\sigma = 0.0551$)</th>
<th>Comp. B ($\sigma = 0.0638$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\xi_0 = 1.10$</td>
<td>$\xi_0 = 1.15$</td>
</tr>
<tr>
<td>0.95</td>
<td>0.16%</td>
<td>0.07%</td>
</tr>
<tr>
<td>0.90</td>
<td>0.18%</td>
<td>0.08%</td>
</tr>
<tr>
<td>0.85</td>
<td>0.18%</td>
<td>0.08%</td>
</tr>
<tr>
<td>0.80</td>
<td>0.18%</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

**References**
