Ambiguity Aversion and Wealth Effects on Demand for Insurance

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Abstract

Risk aversion and decreasing absolute risk aversion play prominent roles in the expected utility theory of demand for insurance against a known risk of accident losses. Demand for insurance against an accident risk that is not known with certainty depends on the attitude toward bearing this ambiguity as well as the underlying risk. Ambiguity aversion in the recursive model developed by Klibanoff et al. (2005) reinforces risk aversion in expected utility, increasing demand for insurance when pricing is not actuarially fair. However, greater ambiguity aversion does not necessarily result in a further increase in demand for insurance. Demand increases when possible accident risks can be ranked by the monotone likelihood ratio order, which is necessarily true for one-loss accidents. Under this ordering, ambiguity aversion also reinforces decreasing absolute risk aversion if preferences satisfy nonincreasing absolute ambiguity aversion. However, it is otherwise possible for insurance to be inferior in the absence of ambiguity, but normal in its presence, and vice versa.

Keywords: Decreasing absolute risk aversion, decreasing absolute ambiguity aversion

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1 Introduction

In expected utility theory, risk aversion and decreasing absolute risk aversion (DARA) play prominent roles in predicting behavior by individuals demanding market insurance against a known risk of accident losses. If insurance pricing is actuarially fair, then risk-averse decision makers demand full coverage. When pricing is unfair but still favorable, only partial coverage is demanded, and demand is lower at higher levels of wealth if risk preferences satisfy DARA, in which case insurance is an inferior good.

These predictions depend on the accident risk being known with certainty. However, situations in which accident risk is not known with precision are surely prevalent, and in these instances of ambiguity, insurance demand depends on the manner in which uncertainty about risk affects decision making. As first reported by Ellsberg (1961), and subsequently confirmed by formal surveys and laboratory experiments, people are typically averse to bearing uncertainty about risk.1 The smooth, recursive model of ambiguity developed by Klibanoff et al. (KMM 2005) provides a useful framework for investigating the implications of ambiguity aversion for risk-averse and DARA behaviors.

In the KMM model, the decision maker’s attitude toward bearing uncertainty about risk is captured in the curvature of a preference functional defined on expected utility, the latter being random because of the decision maker’s uncertainty about the accident risk. When there is no ambiguity, or with ambiguity neutrality, the decision criterion reduces to expected utility. When the decision maker is ambiguity averse, the

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1 See Camerer and Weber (1992) and Etner et al. (2012) for reviews of the relevant literature.
preference functional is strictly concave, and the decision maker is willing to pay a positive amount to avoid uncertainty about expected utility. Thus, as shown by Alary et al. (2013), a risk-averse and ambiguity-averse decision maker is willing to pay more to avoid an ambiguous risk than to avoid the same risk in the absence of ambiguity. As a preliminary, two complementary results are established in this study, the first showing that actuarially fair pricing is necessary and sufficient for decision makers to demand full coverage for an ambiguous risk, and the second showing that, when insurance pricing is unfair, ambiguity aversion increases demand for coverage.

By separating uncertainty about risk from the attitude toward bearing this uncertainty, the KMM model affords the opportunity to analyze the comparative statics of greater aversion toward bearing a given uncertainty about accident risk. Greater ambiguity aversion reduces welfare, just as would a reduction in wealth. However, greater ambiguity aversion has no effect on the strength of aversion toward bearing unambiguous risks, which is governed by DARA when wealth changes. For this reason, a focus on greater ambiguity aversion serves as a useful prelude to the analysis of greater wealth.

Although ambiguity aversion increases demand for insurance, greater ambiguity aversion does not necessarily lead to a further increase in demand. Since the insurance and portfolio choice problems are formally equivalent, the results obtained by Gollier (2011) concerning greater ambiguity aversion for a KMM investor are germane. When the possible return risks for an ambiguous asset belong to the monotone likelihood ratio
(MLR) order, greater ambiguity aversion reduces demand for the ambiguous asset. In the insurance context, if the possible accident risks belong to the MLR order, then greater ambiguity aversion increases insurance demand.

The MLR restriction, however, is not sufficient to determine the comparative statics effect of greater wealth without additional restrictions on preferences beyond risk aversion and ambiguity aversion. First, the effect of wealth on aversion to bearing unambiguous risks must be taken into account, but is of no consequence with constant absolute risk aversion (CARA). Additionally, a change in wealth can affect the degree of ambiguity aversion. This effect too is inactive when preferences satisfy constant absolute ambiguity aversion (CAAA). Nonetheless, an ambiguity-averse decision maker exhibits DARA behavior, and insurance for an ambiguous risk is inferior, because a uniform increase in wealth increases expected utility by a greater margin for higher-probability accidents, dampening the incentive to transfer wealth to accident loss states.

To further develop intuition for these results, observe that insurance demand depends on the relative value of wealth in the accident loss states, which depends in turn on the ratio of the marginal utilities of wealth and the perceived loss likelihood ratios. Ambiguity aversion distorts the objective likelihood ratios in a direction that causes the perceived ratios to overstate the likelihood of accident losses, leading to an increase in insurance demand. The extent of this “pessimistic” distortion depends on the correlation between loss likelihood ratios and the marginal value of higher expected utility. When the possible risks are ranked by first-order stochastic dominance (FSD), expected utility

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2 Gollier presents a numerical example in which greater ambiguity aversion leads to an increase in demand for the ambiguous asset. It follows that greater ambiguity aversion can result in a reduction in demand for insurance against an ambiguous risk.
varies monotonically across the risks, and greater ambiguity aversion increases variation in the marginal value of expected utility. This increased variation is positively correlated with variation in the loss likelihood ratios when the accident risks belong to the MLR subset of the FSD order. As a result, under the MLR ordering, greater ambiguity aversion increases the pessimistic distortion of objective risks, increasing the relative value of wealth in loss states, leading to an increase in demand for insurance.

The comparative statics analysis of greater wealth differs in three respects. First, greater wealth can influence the degree of aversion toward bearing unambiguous risk, as well as variation in the degree of ambiguity aversion across the possible accident risks which influences the perceived loss likelihood ratios. With CARA and CAAA, ambiguity aversion and wealth effects on demand for insurance differ solely because of their distinct effects on the decision maker’s willingness to transfer utility toward higher-probability accidents.

With ambiguity aversion, acceptance sets for the preference functional evaluating expected utility are strictly convex, and become less inclusive with greater ambiguity aversion, increasing the relative value of expected utility in states with higher-probability accidents, leading to an increase in demand for insurance. With CAAA, a change in the wealth endowment that increases expected utility uniformly across accident risks would have no effect on the relative value of expected utility under alternative accident risks. However, a uniform increase in wealth across accident risks increases expected utility by a greater margin for higher-probability accidents, which reduces the relative value of expected utility for these risks, leading to a reduction in demand for insurance.
Thus, CAAA coupled with CARA results in DARA behavior and, in this instance, insurance is inferior due solely to ambiguity aversion. While inferiority is reinforced by DARA and by decreasing ambiguity aversion, increasing ambiguity aversion can reverse the prediction and result in normality for insurance demand.

The expected utility and KMM decision criteria are introduced in the next section. Implications of ambiguity aversion for insurance demand are analyzed in section 3, and comparative statics effects of greater ambiguity aversion and greater wealth are examined in sections 4 and 5, respectively. Predictions derived from the KMM model are contrasted with those implied by alternative models of ambiguity aversion in section 6. Conclusions are offered in the final section.

2 Expected Utility and KMM Utility

Consider an expected utility maximizer endowed with a sure wealth $w$, a strictly risk-averse and differentiable utility function $u(w)$, and an accident risk. The accident state occurs with probability $\theta \in (0,1)$, and a loss $x \in (0,\bar{x}]$ is incurred according to the conditional cumulative probability distribution $P(x \mid \theta)$. Expected utility is given by

$$EU(w, [w(x)] \mid \theta) \equiv (1 - \theta)u(w) + \theta \int u(w - x) \, dP(x \mid \theta),$$

(1)

where $[w(x)]$ denotes contingent wealth for the loss states. Given strict risk aversion, $EU$ is a strictly concave function of state-contingent wealth $(w, [w(x)])$.

If the decision maker does not know the objective accident risk $P(x \mid \bar{\theta})$ with certainty, then the decision criterion is assumed to be the recursive objective function developed by Klibanoff et al. (2005) for decision contexts in which risk is ambiguous.

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3 Limits on integration are omitted to simplify notation.
With the possible accident risks \( \{P(x \mid \theta)\} \) ranked by the probability of accident \( \theta \), let \( F(\theta) \) represent the decision maker’s beliefs concerning which of these risks governs the loss probabilities. The decision criterion is the expected value of a preference functional \( \varphi \) defined on expected utility,\(^4\)

\[
AEU(w,[w(x)]) = \varphi^{-1}\left(\int \varphi\left(EU(w,[w(x)] \mid \theta)\right)dF(\theta)\right).
\]

When \( \varphi \) is linear, and therefore ambiguity neutral, the decision maker reduces the compound lottery to its one-stage equivalent. Assuming that \( F \) is unbiased, the expected risk is equal to the objective risk and, with ambiguity neutrality, \( AEU(w,[w(x)]) \) then reduces to expected utility, \( EU(w,[w(x)] \mid \bar{\theta}). \(^5\) When the decision maker is ambiguity averse, the preference functional \( \varphi \) is strictly concave.

**Lemma 1**\(^6\) If \( \varphi \) is ambiguity neutral or ambiguity averse, then

\[
\int \varphi(EU(w,[w(x)] \mid \theta))dF(\theta)
\]

is a strictly concave function of state-contingent wealth \( (w,[w(x)]) \).

### 3 Ambiguity Aversion and Insurance Demand

Being strictly concave, both \( EU \) and \( \int \varphi dF \) are strictly quasiconcave, and therefore have strictly convex upper contour sets, which constitute their Yaari (1969)

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\(^4\) Several other developments of subjective ambiguity preference yield the smooth, recursive decision criterion (2), including those introduced by Nau (2006), Chew and Sagi (2008), Ergin and Gul (2009), and Nielson (2010).

\(^5\) The assumption that \( F \) is unbiased ensures that introducing ambiguity has no effect on the behavior of an ambiguity-neutral decision maker.

\(^6\) Proofs for the Lemmas are provided in the appendix.
acceptance sets.\(^7\) The next result shows that, with risk aversion and ambiguity aversion, the acceptance sets for \( AEU \),

\[
S^{AEU} = \{ (w, [w(x)]) \mid AEU(w, [w(x)]) \geq u(\hat{w}) \},
\]  

are nested inside those for \( EU \),

\[
S^{EU} = \{ (w, [w(x)]) \mid EU(w, [w(x)]) \geq u(\hat{w}) \},
\]  
as illustrated in Figure 1. The proof involves a comparison of the marginal rates of substitution for expected utility,

\[
MRS^{EU} = \frac{\overline{\theta} p(x \mid \overline{\theta}) u'(w-x)}{1 - \overline{\theta}} u'(w),
\]  
with those for ambiguous expected utility,

\[
MRS^{AEU} = \frac{\varphi' \cdot \theta p(x \mid \theta) dF u'(w-x)}{\int \varphi' \cdot (1 - \theta) dF u'(w)},
\]

where \( p(x \mid \theta) \) is the probability density for \( P(x \mid \theta) \).\(^8\)

**Lemma 2** If \( \varphi \) is ambiguity averse, then the acceptance sets \( S^{AEU} \) are nested within the acceptance sets \( S^{EU} \), with tangency \( MRS^{AEU} = MRS^{EU} \) when \( x = 0 \), and single crossing \( MRS^{AEU} > MRS^{EU} \) when \( x > 0 \).

The nesting of acceptance sets established in Lemma 2 indicates the direction of distortion in the loss likelihood ratios caused by ambiguity aversion. Specifically, in every loss state, the perceived likelihood ratio is greater than the objective ratio,

\(^7\) The positive monotonicity of \( \varphi^{-1} \) implies that the acceptance sets for \( AEU \) coincide with those for \( \{ \varphi dF \}. \)

\(^8\) Throughout we use primes denote derivatives for univariate functions.
\[
\frac{\int \varphi' \cdot \vartheta \ p(x \mid \vartheta) \ dF}{\int \varphi' \cdot (1 - \vartheta) \ dF} > \frac{\bar{\vartheta} \ p(x \mid \bar{\vartheta})}{1 - \bar{\vartheta}}.
\]

This “pessimism” property of ambiguity preferences determines the effect of ambiguity aversion on insurance demand when pricing is favorable but actuarially unfair.

Insurance offers the opportunity to transfer wealth to the accident loss states. The demand for insurance can be represented by the demands for wealth in loss states given objectively fair prices augmented by a nonnegative proportional loading factor \( \lambda \), \(^9\)

\[
\pi(x) = \bar{\vartheta} p(x \mid \bar{\vartheta})(1 + \lambda).
\]

The price of wealth in the no-accident state is \( \pi(0) = 1 - \bar{\vartheta}(1 + \lambda) \), and the budget constraint limits total spending on wealth to the value of the endowment, \( \pi(0)w + \int \pi(x)(w - x) \, dx \). Using \( w^*(x) \) to denote optimal state-contingent wealth demands, coverage is given by \( c(x) = x - [w^*(0) - w^*(x)] \) in each of the accident loss states, and the insurance premium is \( m = \int \pi(x)c(x) \, dx \), so that \( w^*(0) = w - m \).\(^{10}\) The first-order conditions imply equality between the marginal rates of substitution

\[
MRS_{AEU} = \frac{\int \varphi' \cdot \vartheta \ p(x \mid \vartheta) \ dF \ u'(w^*(x))}{\int \varphi' \cdot (1 - \vartheta) \ dF \ u'(w^*(0))},
\]

and the corresponding price ratios

\[
\frac{\pi(x)}{1 - \bar{\vartheta}(1 + \lambda)} = \frac{\bar{\vartheta} p(x \mid \bar{\vartheta})(1 + \lambda)}{1 - \bar{\vartheta}(1 + \lambda)}.
\]

**Lemma 3** Full coverage is optimal if and only if \( \lambda = 0 \).

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\(^9\) The insurer is assumed to be risk neutral and ambiguity neutral, and is constrained to zero expected profit after incurring indemnification costs and other variable costs accounted for by \( \lambda \).

\(^{10}\) Adding a fixed loading charge to the premium would not affect the results.
Lemma 3 shows that all losses are fully covered if and only if the price ratios are undistorted by proportional loading. Henceforth, assume that the loading factor $\lambda$ is positive so that full insurance is not optimal for any loss. The nesting of acceptance sets established in Lemma 2 then implies that ambiguity aversion increases positive demands for insurance coverage.

4 Comparative Statics: Greater Ambiguity Aversion

The necessary and sufficient condition for insurance demand to increase with greater ambiguity aversion is that perceived loss likelihood ratios increase. To evaluate the comparative statics of greater ambiguity aversion, we follow Klibanoff et al. and identify one preference functional as being more ambiguity averse than a second if the first is a concave transformation of the second. Thus, greater ambiguity aversion increases insurance demand if the perceived loss likelihood ratios increase when $\varphi$ is replaced by $G(\varphi)$ for an increasing, strictly concave function $G$.

Henceforth, let wealth $w$ represent the endowed wealth net of insurance premium, with $x$ representing an uninsured loss amount, so that accident risks $\{P(x | \theta)\}$ now reflect exposure to uninsured losses.\(^{11}\) Greater ambiguity aversion has a sign-definite effect on the distorted likelihood ratios if the possible accident risks belong to the MLR order. This order includes those for which the unconditional loss probabilities increase in

\[^{11}\text{Since exposure to uninsured losses depends on the demand for coverage, accident risks after insurance may belong the MLR order even if the uninsured risks do not, except in the case of one-loss accidents which trivially belong to the MLR order.}\]
every loss state, or when the conditional probabilities of loss are independent of the probability of accident, or there is only one loss state.

Two implications of the MLR ordering are exploited. First, the likelihood ratio for each accident loss, \( \theta p(x \mid \theta) / (1 - \theta) \), is an increasing function of the probability of accident, and second, a FSD shift induced by a greater accident probability reduces expected utility.

**Proposition 1** Assume positive proportional loading and positive insurance demand by the AEU criterion. An increase in ambiguity aversion increases insurance demand if possible accident risks belong to the MLR order.

**Proof** The necessary and sufficient condition for insurance demand to increase is that \( MRs^{AEU} \) increases with greater ambiguity aversion, which requires that the perceived loss likelihood ratios increase. Replacing \( \varphi \) with a concave transformation \( G(\varphi) \) in equation (9) for \( MRs^{AEU} \), one finds that greater ambiguity aversion increases the perceived loss likelihood ratios if

\[
\frac{\int G' \varphi' \cdot \theta p(x \mid \theta) dF}{\int G' \varphi' \cdot (1 - \theta) dF} > \frac{\int \varphi' \cdot \theta p(x \mid \theta) dF}{\int \varphi' \cdot (1 - \theta) dF},
\]

or equivalently, if

\[
\int G' \varphi' \cdot (1 - \theta) \cdot T(\theta) dF > 0,
\]

where

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12 Chiu (2005) likens a change in which all unconditional loss probabilities decline to the installation of a burglar alarm: the probability of losing nothing increases and the probability of losing each positive amount declines.

13 Alary et al. (2013) show that a deductible contract is optimal for the KMM criterion when the conditional loss probabilities are independent of the probability of accident.
\[ T(\theta) = \left[ \frac{\theta p(x | \theta)}{1 - \theta} - \frac{\phi' \cdot \theta p(x | \theta) dF}{\phi' \cdot (1 - \theta) dF} \right]. \]

The expected value of the integral on the left-hand side of inequality (11) is zero when \( G' \) is a constant, while the MLR ordering ensures that \( T(\theta) \) is increasing in \( \theta \). Hence, \( T(\theta) \) must change sign from negative to positive at some value \( \hat{\theta} \) as \( \theta \) increases.

Using \( \hat{G}' \) to denote the value of \( G' \) at \( \hat{\theta} \), we substitute \( \hat{G}' \phi' \cdot T(\theta) dF = 0 \) for the right-hand side of inequality (11) to obtain

\[ \int (G' - \hat{G}') \phi' \cdot (1 - \theta)T(\theta) dF > 0. \] (12)

The effect of an increase in \( \theta \) on \( G' \) is given by \( G^* \phi' \cdot \partial EU / \partial \theta \), which is positive since \( \partial EU / \partial \theta \) is negative under the MLR ordering. Hence, \( G' - \hat{G}' \) changes sign from negative to positive at \( \hat{\theta} \) as \( \theta \) increases. It follows that the integrand on the left-hand side of inequality (12) is equal to zero at \( \hat{\theta} \) and is positive otherwise. ■

No restrictions on preferences beyond risk aversion and ambiguity aversion are exploited in proving Proposition 1. The degree of ambiguity aversion has no effect on the ratio of marginal utilities, while its influence on the perceived likelihood ratios is channeled through the degree of concavity for the preference functional \( \phi \). When possible accident risks belong to the FSD order, the change in the marginal value of expected utility occasioned by greater ambiguity aversion is monotonically increasing with respect to accident probability, while the likelihood ratios are also positive monotonic when accident risks belong to the MLR suborder. The positive correlation between these two effects increases the relative value of wealth in loss states, leading to an increase in demand for insurance.
The driving force behind Proposition 1 is revealed by focusing on a one-loss accident and supposing that only two accident probabilities are possible. In this decision environment, the tradeoff in ambiguity preferences between expected utility under the two accident probabilities determines the degree of distortion in the perceived loss likelihood ratio, and the degree of distortion increases as willingness to transfer utility to the high-probability accident increases.\textsuperscript{14}

Figure 2 illustrates indifference contours for $\int \varphi (EU (\cdot | \theta ))dF$ and for $\int G(\varphi (EU (\cdot | \theta )))dF$, assuming that $\varphi$ and $G$ are both strictly concave. The acceptance sets for $\int G(\varphi )dF$ are nested within those for $\int \varphi dF$ as illustrated, so that with partial coverage, as at point A, the decision maker’s willingness to transfer utility to the high-probability accident is greater under the more concave preference functional, resulting in a greater distortion of the loss likelihood ratio and an increase in demand for insurance.

5 Comparative Statics: Greater Wealth

A decision criterion exhibits DARA behavior when a uniform increase in wealth across states reduces aversion to bearing risk, thereby leading to greater risk taking. In the case of $AEU$, the risk in question is ambiguous, whereas for $EU$ the risk is unambiguous. Nonetheless, for both $EU$ and $AEU$, the property of DARA behavior is indicated when acceptance sets $S^{EU}$ and $S^{AEU}$ become more inclusive as wealth increases uniformly across states. For this to occur, the marginal rates of substitution must decline as wealth increases.

\textsuperscript{14} Using $f_i$ to denote the subjective probability of accident risk $\theta_i$, the perceived likelihood of loss is $\theta_1 f_1 + \theta_2 f_2 M$ divided by $(1 - \theta_1) f_1 + (1 - \theta_2) f_2 M$, where $M = \varphi'(EU (\cdot | \theta_2 ))/\varphi'(EU (\cdot | \theta_1 ))$ measures the willingness to transfer utility to accident risk $\theta_2$. The perceived loss likelihood is an increasing function of $M$ when $\theta_2 > \theta_1$. 
For $EU$ one obtains from equation (5)

$$\partial MRS^EU / \partial w = MRS^EU [R^{EU} (w) - R^{EU} (w - x)],$$

(13)

where $R^{EU} = -u''(w)/u'(w)$ is the Arrow-Pratt index of risk aversion. Equation (13) is negative if and only if $EU$ exhibits DARA. For $AEU$ one obtains from equation (6)

$$\partial MRS^{AEU} / \partial w = MRS^{AEU} [R^{AEU} (w) - R^{AEU} (w - x) + \Omega],$$

(14)

where

$$\Omega = \frac{\int \phi'' \cdot EU_w \cdot \theta \ p(x \mid \theta) dF}{\int \phi' \cdot \theta \ p(x \mid \theta) dF} - \frac{\int \phi'' \cdot EU_w \cdot (1 - \theta) dF}{\int \phi' \cdot (1 - \theta) dF},$$

(15)

and $EU_w = \partial EU(w, [w(x)] \mid \theta) / \partial w$ denotes the expected marginal utility of wealth given accident risk $P(x \mid \theta)$. A comparison of equations (13) and (14) reveals that ambiguity aversion adds $\Omega$ to the effect of greater wealth on aversion toward bearing unambiguous risks, where $\Omega$ captures the effect of greater wealth on the perceived loss likelihood ratios.

Multiplying equation (15) by $\int \phi' \cdot \theta \ p(x \mid \theta) dF$ yields

$$\Omega \cdot \int \phi' \cdot \theta \ p(x \mid \theta) dF$$

$$= -\int R^\phi \cdot EU_w \cdot \phi' \cdot (1 - \theta) T(\theta) dF$$

(16)

after multiplying and dividing $\phi''$ by $\phi'$, where the index $R^\phi = -\phi''/\phi'$ measures the degree of absolute ambiguity aversion. Equation (16) reveals that the sign of $\Omega$ depends on the stochastic ordering of possible accident risks through variation in $T(\theta)$, as well as variation in the expected marginal utility of wealth $EU_w$, and variation in the degree of absolute ambiguity aversion $R^\phi$. By analogy with expected utility theory, a preference functional $\Phi$ satisfies nonincreasing absolute ambiguity aversion (NIAAA) when the
degree of ambiguity aversion measured by $R^\varphi$ declines or remains unchanged as expected utility increases.

**Proposition 2**  $AEU$ exhibits DARA behavior if possible accident risks belong to the MLR order, $u$ satisfies CARA or DARA, and $\varphi$ is ambiguity averse and satisfies NIAAA.

**Proof**  $AEU$ exhibits DARA behavior if and only if equation (14) for the effect of greater wealth on $MRS^{AEU}$ is negative. Since the difference $R^{EU}(w) - R^{EU}(w - x)$ is nonpositive with nonincreasing absolute risk aversion, it suffices to show that $\Omega$ is negative, which is equivalent to establishing that the right-hand side of equation (16) is negative.

Under the MLR ordering, $T(\theta)$ changes sign from negative to positive at $\hat{\theta}$ as $\theta$ increases. For the accident risk $P(x \mid \hat{\theta})$, let $E\hat{U}_w = \hat{\partial}EU(w, [w(x)] \mid \hat{\theta}) / \hat{\partial}w$ denote the expected marginal utility of wealth and let $\hat{R}^\varphi$ denote the index of absolute ambiguity aversion. After replacing $EU_w$ with $E\hat{U}_w + (EU_w - E\hat{U}_w)$ and $R^\varphi$ with $\hat{R}^\varphi + (R^\varphi - \hat{R}^\varphi)$, equation (16) can be written as

$$\Omega \cdot \{ \varphi' \cdot \theta \ p(x \mid \theta) dF = -\hat{R}^\varphi \cdot \{ \varphi' \cdot (EU_w - E\hat{U}_w) \cdot (1 - \theta)T(\theta) dF$$

$$- \{ (R^\varphi - \hat{R}^\varphi) \cdot EU_w \cdot \varphi' \cdot (1 - \theta)T(\theta) dF \}
$$

after grouping like terms.\(^{15}\)

Since the accident risks belong to the FSD order, the expected marginal utility of wealth is an increasing function of the accident probability, given risk aversion. Hence,

\(^{15}\) The term involving $E\hat{U}_w$ vanishes, since the expected value of $\varphi' \cdot (1 - \theta)T(\theta)$ is zero.
both $T(\theta)$ and $EU_w - \hat{EU}_w$ change sign from negative to positive at $\hat{\theta}$ as $\theta$ increases, so the first integral on the right-hand side of equation (17) is positive. With CAAA, there is no variation in the degree of ambiguity aversion from $\hat{R}^\varphi$ and the second term on the right-hand side of equation (17) vanishes, in which case $\Omega$ is negative.

With decreasing absolute ambiguity aversion, $R^\varphi - \hat{R}^\varphi$ is a decreasing function of expected utility, while expected utility itself declines as $\theta$ increases since the accident risks belong to the FSD order. Thus, $R^\varphi - \hat{R}^\varphi$ changes sign from negative to positive at $\hat{\theta}$ as $\theta$ increases, and the second integral on the right-hand side of equation (17) is positive. Hence, $\Omega$ is negative when possible accident risks belong to the MLR order and $\varphi$ satisfies NIAAA. ■

Proposition 2 shows that $AEU$ exhibits DARA behavior if the utility function $u$ satisfies CARA, the preference functional $\varphi$ satisfies CAAA, and possible accident risks belong to the MLR order. Some insight is gained by again focusing on a one-loss accident with two possible accident probabilities. As noted earlier, the tradeoff in ambiguity preferences determines the degree of distortion in the perceived loss likelihood ratio, and the degree of distortion increases as willingness to transfer utility to the high-probability accident increases.

With CAAA, the tradeoff between accident risks remains constant when expected utility increases by the same amount for both accident risks, as illustrated in Figure 3 by the change from A to B. However, a uniform increase in wealth increases expected utility by a greater margin for the high-probability accident, resulting in the change from A to C in Figure 3, and the rate of substitution between the accident risks declines. Thus,
with CAAA, greater wealth reduces willingness to transfer utility to the high-probability accident, reducing the perceived likelihood of accident and the demand for insurance.

Proposition 2 identifies conditions under which ambiguity increases the strength of DARA for an expected utility maximizer: If possible accident risks belong to the MLR order, then ambiguity preferences that satisfy NIIAA strengthen DARA behavior. Under these conditions, insurance is an inferior good for both EU and AEU. Proposition 2 also implies that insurance could be normal in the absence of ambiguity, but inferior in its presence.

Finally, if \( u \) satisfies CARA and \( \phi \) satisfies increasing absolute ambiguity aversion (IAAA), an ambiguity-averse decision maker may still exhibit DARA behavior, but could instead exhibit behavior consistent with increasing absolute risk aversion (IARA). With IAAA, \( R^{\phi} - \hat{R}^{\phi} \) is a decreasing rather than increasing function of \( \theta \), and the two terms on the right-hand side of equation (17) then have opposite signs. If the strength of IAAA is great enough relative to the degree of absolute ambiguity aversion at \( \hat{\theta} \), then the second term dominates the first and AEU exhibits IARA behavior. Thus insurance can be inferior for EU but normal for AEU if IAAA is sufficiently strong.

6 Ambiguity Aversion and Wealth Effects in Alternative Models of Ambiguity

The AEU criterion satisfies second-order risk aversion in the sense of Segal and Spivak (1990), as its indifference contours are smoothly differentiable throughout. Hence, the model predicts that ambiguity aversion increases demand for insurance when insurance pricing is unfair, but never leads to a demand for full coverage. This last prediction does not carry over to the alternative, non-recursive models of ambiguity, rank dependent (Quiggin (1982)), Choque (Schmeidler (1989)), maxmin (Gilboa and
Schmeidler (1989)), and $\alpha$-maxmin (Ghirardato et al. (2004)) expected utility. These models satisfy first-order risk aversion and have kinked indifference curves at full coverage. As a result, in these models, ambiguity aversion can result in demand for full coverage when insurance pricing is unfair.\(^{16}\)

The non-recursive models yield similar decision criteria, but only the $\alpha$-maxmin model developed by Ghirardato et al. (2004) admit a distinction between ambiguity and ambiguity preferences. Assume that $\theta_1$ and $\theta_2$ are the best and the worst accident risks, respectively, so that

$$EU(w,[w(x)]|\theta_1) \geq EU(w,[w(x)]|\theta) \geq EU(w,[w(x)]|\theta_2)$$

for all possible accident risks $\{\theta, P(x|\theta)\}$. The $\alpha$-maxmin decision criterion is then

$$\alpha EU \equiv \alpha EU (w,w-x|\theta_1) + (1-\alpha)EU (w,w-x|\theta_2),$$

(18)

where $1-\alpha$ captures the degree of pessimism as $\theta_2$ is the worst accident. The decision maker is ambiguity neutral when $\alpha$ equals one half. When $\alpha$ equals zero, $\alpha EU$ is the maxmin decision criterion, the most ambiguity averse in the class.\(^{17}\)

The marginal rates of substitution for $\alpha EU$ are

$$MRS_{\alpha EU} = \frac{\alpha \theta_1 + (1-\alpha)\theta_2}{\alpha(1-\theta_1) + (1-\alpha)(1-\theta_2)} \frac{u'(w-x)}{u'(w)}.$$  

(19)

As with the KMM criterion, the perceived likelihood ratio is pessimistic with ambiguity

\(^{16}\) For example, with two possible probabilities for a one-loss accident, $\theta_2 > \theta_1$, the maxmin criterion uses the less favorable probability, which is $\theta_2$ when the loss $x$ is positive, resulting in $MRS = \theta_2 u'(w-x)/(1-\theta_2)u'(w)$. With $\lambda = (\theta_2 - \bar{\theta}) / \bar{\theta}$, positive loading leads to partial coverage for the KMM criterion, but the loaded likelihood ratio equals $\theta_2/(1-\theta_2)$ and the maxmin criterion demands full coverage.

\(^{17}\) The maxmin criterion is $EU(w,w-x|\theta_2)$ since $\theta_2$ is the least favorable probability from the perspective of $EU$. 

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aversion, when $\alpha$ is below one half. However, unlike the KMM criterion, marginal rates of substitution depend only on two accident risks regardless of the stochastic ordering of the possible risks, including $\theta_1$ and $\theta_2$. Thus, greater ambiguity aversion, indicated by a smaller value for $\alpha$, always increases the pessimistic distortion associated with ambiguity in the $\alpha$-maxmin model, whereas this is guaranteed in the smooth, recursive KMM model only when accident risks belong to the MLR order.

Equation (19) also reveals a property shared by all of the non-recursive models, namely that greater wealth has no effect on perceived likelihood ratios. As a result, DARA behavior is dictated solely by DARA in the attitude toward bearing unambiguous risks, in contrast with the smooth, recursive KMM criterion where the prediction of DARA behavior relies on the MLR ordering of accident risks as well as nonincreasing absolute ambiguity aversion.

7 Conclusions

The smooth, recursive model of ambiguity predicts that an ambiguity-averse, expected utility maximizer demands more insurance when the accident risk is ambiguous rather than unambiguous, given favorable but actuarily unfair insurance pricing. Sign-definite comparative statics predictions for greater wealth and greater ambiguity aversion are obtained when possible accident risks belong to the MLR order. This restriction alone suffices for insurance demand to increase with greater ambiguity aversion. The individual exhibits DARA behavior and reduces demand for insurance against an ambiguous accident risk as wealth increases if the ambiguous risk belongs to the MLR order and risk preferences satisfy nonincreasing absolute risk aversion and ambiguity preferences satisfy nonincreasing absolute ambiguity aversion.
The results on normality for insurance demand apply equally to portfolio choice. A KMM investor exhibits DARA behavior with respect to demand for an ambiguous asset belonging to the MLR order if risk preferences satisfy nonincreasing absolute risk aversion and ambiguity preferences satisfy nonincreasing absolute ambiguity aversion. In these instances, demand for the ambiguous asset is normal. With constant absolute risk aversion, demand for an unambiguous asset and demand for insurance against an unambiguous accident risk are both neither normal nor inferior, yet these demands can be either normal or inferior in the presence of ambiguity if the decision maker is ambiguity averse.
Appendix

Proof of Lemma 1  Strict risk aversion implies the strict concavity of $EU(w,[w(x)]|\theta)$ in state contingent wealth, which is inherited by $\varphi(EU(w,[w(x)]|\theta))$ when $\varphi$ is either linear or concave. Its expected value, $[\varphi(EU(w,[w(x)]|\theta))dF(\theta)$, is then also a strictly concave function of state-contingent wealth for the same reason that expected utility is strictly concave. ■

Proof of Lemma 2  Observe that, when wealth is the same in every state, both marginal rates of substitution (5) and (6) reduce to the objective likelihood of accident loss $x$, $\bar{\theta} p(x|\bar{\theta})/(1-\bar{\theta})$, since $\varphi'$ is then independent of $\theta$ and the marginal utility ratios equal one. Hence, indifference contours for $EU$ and $AEU$ are tangent at fully insured positions.

Alary et al. (2013) show that, as a consequence of the pessimism introduced with ambiguity aversion, the risk premium $\pi^{AEU}$ for ambiguous expected utility, defined implicitly by the relation

$$AEU(w,[w(x)]) = u(\bar{w} - \pi^{AEU}) ,$$

is greater than the Arrow-Pratt risk premium $\pi^{EU}$, defined implicitly by the relation

$$EU(w,[w(x)]|\bar{\theta}) = u(\bar{w} - \pi^{EU}) ,$$

where $\bar{w}$ denotes expected wealth. It follows that, given a sure wealth $\hat{w}$, every state-contingent wealth acceptable to $AEU$ is acceptable to $EU$, while some state-contingent wealth acceptable to $EU$ is not acceptable to $AEU$. Since by Lemma 1 the acceptance
sets are strictly convex, \( MRS^{AEU} \) must exceed \( MRS^{EU} \) in positive loss states. Hence,

\[ S^{AEU} \subset S^{EU} \]  

as illustrated in Figure 1. ■

Proof of Lemma 3  With full coverage, expected utility does not vary with \( \theta \) and \( MRS^{AEU} \) reduces to the undistorted likelihood ratio, \( \bar{\theta}p(x | \bar{\theta})/(1 - \bar{\theta}) \), at every positive loss \( x \). Lemma 1 implies that this occurs only with full coverage. It follows that full coverage is consistent with equality between \( MRS^{AEU} \) and the price ratio at every positive loss if and only if \( \lambda = 0 \). ■
References


Figure 1: $AEU$ vs $EU$
Figure 2: Greater ambiguity aversion with $\theta_2 > \theta_1$
Figure 2: CAAA with $\theta_2 > \theta_1$