Reinsurance Networks and Their Impact on Reinsurance Decisions: Theory and Empirical Evidence

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ABSTRACT

This paper investigates the role of reinsurance networks in an insurer’s reinsurance purchase decision. Drawing on network theory, we develop a framework that delineates how the pattern of linkages among reinsurers determines three reinsurance costs (loadings, contagion costs, and search and monitoring costs) and characterizes an insurer’s optimal network structure. Consistent with empirical evidence based on longitudinal data from the U.S. property and casualty insurance industry, our model predicts an inverted U-shaped relationship between the insurer’s optimal percentage of reinsurance ceded and the number of its reinsurers. Moreover, we find that a linked network may be optimal ex ante even though linkages among reinsurers may spread financial contagion, supporting the model’s prediction regarding social capital benefits associated with network cohesion. Our theoretical model and empirical results have implications for other networks such as loan sale market networks and over-the-counter dealer networks.

Keywords: Reinsurance Costs, Optimal Network Structure, Network Centrality, Network Cohesion
1. Introduction

The real-world prominence of reinsurance as a risk transfer method in the insurance industry raises questions about its benefits and costs. While there exists a rich literature that explains why insurers should purchase reinsurance (Mayers and Smith 1982, Cole and McCullough 2006, Cole et al. 2011), little is known about how and the extent to which varied reinsurance costs affect such organizational activities. Much of the prior research makes the implicit but extreme assumption that reinsurance costs are exogenous or independent of risk characteristics and ignores costs associated with the distinct risk exchange relations between insurers and reinsurers. Studies have so far linked reinsurance demand to firm characteristics, such as firm size, group affiliation and organizational form (Cole and McCullough 2006, Cole et al. 2011). While researchers have also looked at how reinsurance demand relates to exogenous reinsurance costs, many of them ignore determinants of those costs due to the lack of a promising venue in which to quantify reinsurance costs, which severely limits previous studies in this area. To fill the gap, this paper brings a network perspective to the study of reinsurance costs, motivated by the proliferation of reinsurance networks. Building on a well-known optimizing model of costly external financing, we directly link various reinsurance costs to an insurer’s network structure. Our evidence confirms the conjecture that an insurer’s network relationships are significant determinants of its level of reinsurance purchase.

Networks are a common market phenomenon (Hochberg et al. 2007, Yu et al. 2011). Among assorted networks, reinsurance networks are not entirely new, but have been evolving rapidly in number, form and complexity in the relatively short period of a half century. In the insurance industry, insurers tend to be closely tied to multiple reinsurers to transfer business risk (Garven and Grace 2007). This indicates that insurers are bound by their current and past risk management practices into webs of relationships with those partners. Hence, their use of reinsurance is influenced by direct and indirect ties among firms embedded in networks.

In light of the above observations, we present a simple model of an insurer’s reinsurance demand in the context of a networked market following Froot et al. (1993). Froot et al.’s (1993) framework is built on the pecking order theory of financing and the correlation between investment opportunities and risk factors being hedged, both of which are satisfied in our insurance setting. First, the pecking order theory
of financing has been applied to an intermediated market such as an insurance market in the existing literature because “intermediaries have limited capital and face costs of adding more, as in Froot and Stein (1998). Intermediary costs of external finance would seem natural since intermediaries are themselves corporations, subject to the same kinds of frictions that make corporate hedging desirable in the first place” (Froot and O’Connell, 2008). Second, property and casualty insurers have valuable opportunities to write more policies or purchase policies from other insurers during catastrophic periods when claims by insured are high. Prices are usually high in the aftermath of major catastrophes driven by decreases in the supply of insurance as well as increases in demand. This provides profitable opportunities to those property and casualty insurers that have internal funds to write more business, which makes hedging desirable.

In our model, because of costly external financing (Froot et al. 1993), reinsurance is meaningful and can assist in maximizing the insurer’s value, the mechanism of which, however, is subject to various reinsurance costs—loadings, contagion costs, and search and monitoring costs. Given this setup, we solve for the insurer’s optimal reinsurance ratio. The approach of this study is unique in that whereas research on reinsurance often rests on a neglect of network and community structures with an assumption of independent, atomistic market players, this paper goes deeper and describes how factors related to an insurer’s reinsurance network affect the magnitude of its reinsurance costs, which in turn determine its reinsurance ratio. We provide economic intuition about what generates differences in insurers’ network structures and how their endogenous network planning relates to their levels of reinsurance activities.

Our arguments about various reinsurance costs are built on two properties of an insurer’s network, network centrality and network cohesion. In this study, network centrality refers to the number of reinsurers in the insurer’s network, and network cohesion denotes the strength and cohesiveness of linkages among the reinsurers. Two testable predictions emerge from our theory. First, other things equal, there exists an inverted-U shaped relationship between the optimal reinsurance level and the network centrality. This nonlinear relationship reflects the tradeoff between two effects: (i) transferring risk to more reinsurers decreases the total loadings for the insurer; (ii) transferring risk to more reinsurers increases the insurer’s search and monitoring costs. As the number of reinsurers increases, we show the cost reduction from the first effect exceeds the cost increment from the second effect, but only up to a point. The model thus predicts that the insurer’s optimal reinsurance level first increases with the number of its reinsurers but
there is a cutoff point above which ceding risk to more reinsurers decreases the optimal reinsurance level. Second, the model predicts that the insurer’s reinsurance level has an inverted-U shaped relationship with the network cohesion. This is because network cohesion has two opposite effects on the percentage of reinsurance ceded. On the one hand, an increase in the network cohesion coincides with an enlarged contagion cost, which reduces the reinsurance ratio. On the other hand, an increase in the network cohesion among reinsurers lowers search and monitoring costs because of the benefit of social capital.

We test these predictions based on reinsurance demand from a sample of U.S. property and casualty primary insurers. We construct measures of network centrality and test our first hypothesis on an insurer’s reinsurance levels and the number of reinsurers. We calculate measures of network cohesion and test our second hypothesis on an insurer’s reinsurance level and the linkages among reinsurers. Using a sample of 1,262 primary insurers between 1993 and 2005, in addition to the results consistent with the existing reinsurance literature, we present new evidence on how network resources determine reinsurance levels, which is strongly supportive of the model’s two main predictions. Importantly, the magnitude of the network effect on the reinsurance decision is statistically and economically significant. We show that, on average, transferring risk to one more reinsurer increases an insurer’s risk ceding level by 2%. Moreover, in this market, the benefit of social capital in most cases dominates the cost of contagion among reinsurers. For an average firm, a one standard deviation increase in network cohesion is associated with an 18.8% increase in risk transfer, all else equal.

Our paper contributes to the growing body of research on corporate risk management in general and reinsurance specifically in three ways. First, it complements previous reinsurance studies by shedding new light on reinsurance demand where networks are a common feature. In the literature, it is typically assumed that risk management is a decision made by firms based on their own investment and financing conditions. For example, it has been proposed that firms including insurance companies undertake risk management to increase tax benefits and debt capacity (e.g., Graham and Rogers 2002, Mian 1996, Nance et al. 1993, Smith and Stulz 1985, Cummins et al. 2001), to alleviate bondholders’ concern of underinvestment problems (e.g., Knopf et al. 2002, Mayers and Smith 1987, Mian 1996, Myers 1977), to reduce reliance on costly external finance (e.g., Froot et al. 1993), and to lower the expected costs of financial distress (e.g., Mayers and Smith 1982, Smith and Stulz 1985). In this paper, rather than focusing only on insurers themselves, we focus on the interactions between insurers and a set of reinsurers as well
as linkages among reinsurers. To the best of our knowledge, this paper provides the first theoretical model and direct empirical evidence that an insurer’s optimal reinsurance ratio is a function of the number of its ties with reinsurers and the network cohesion among its reinsurers.

Second, our paper is also related to the broader line of research on optimal hedge ratios (e.g., Brown and Toft 2002, Graham and Rogers 2002, Haushalter 2000). It is well known that in a Modigliani and Miller world, risk management is unnecessary for firms because investors are able to fully diversify risks by using tools available to them. However, corporate risk management becomes relevant when the market is imperfect. Froot et al. (1993) explicitly point out that the need for risk management under an imperfect market does not imply that firms have to hedge completely. This insight leads to an important question in the insurance industry: What are the major factors that determine an insurer’s optimal reinsurance ratio? Drawing on the network perspective, this paper adds to the insurance literature by introducing a significant factor—reinsurance costs—that has been largely ignored or oversimplified in prior research.

Third, our study suggests a promising framework for analyzing risk management networks for some non-insurance sectors such as the banking industry and “over the counter” (OTC) markets. In corporate loan markets, credit risk and liquidity risk shape banking risk management transactions against financial distress. A bank can lay off credit risk by selling loans. Cebenoyan and Strahan (2004) note that banks are active in participating in loan sale market networks for hedging purposes. The structure of the networks should be an important consideration in the banks’ hedging decisions. On the other hand, a bank can transfer credit risk by buying credit insurance through a credit default swap (CDS) in OTC. Similar to regular insurance, CDS contracts are viewed as a mechanism to spread risks. However, too much risk sharing in the CDS market can actually create big problems (Buchanan 2011). Dense webs of CDS sellers make it easy for distress to spread like a virus. Our model can be employed to show that the rising connectivity in the CDS market is a danger when the contagion cost exceeds the social capital benefit, thus providing a venue in which to shed light on the financial crisis in the late 2000’s. Moreover, our model can be applied to other OTC market networks such as municipal bond dealer networks (Li and Schürhoff 2012). All end-users who seek OTC products transact with dealers. Many dealers are closely connected by transacting heavily with each other to hedge the risks from their customer trades (Buchanan 2011). Our study can be used, for example, for a dealer to determine the optimal number of counterparties and network cohesion for risk transfer.
The remainder of the paper is organized as follows. The next section presents our model. In Section 3, we analyze one common risk management network in which reinsurers are linked to some or all other reinsurers, and characterize its optimal solutions. Section 4 is dedicated to the empirical study. We first describe the data, define the variables, and explain the methodology. Then we empirically test the main predictions of the model. Finally, Section 5 concludes.

2. Model

In this section, we first outline the basic model and then review the network literature in the reinsurance context, followed by an introduction of some network measures for reinsurance network analysis.

2.1. Timing

In an economy with four dates, \( t = -1, 0, 1, 2 \), there are two classes of agents: an insurer and a large pool of \( N \) competitive reinsurers that are identical ex ante. Both the insurer and the reinsurers are rational decision makers and for simplicity, we assume neither of them discounts future cash flows.

At date \( t = 0 \), the insurer has an inherited level of net internal assets \( a_0 \) with a random one-period payoff \( \bar{r} \), subject to uncertain shocks such as catastrophe losses. For simplicity, we assume \( \bar{r} \) is normally distributed with a mean of \( \mu = 1 \) and a variance of \( \sigma^2 \). At this moment, the insurer decides to manage risk and must determine the fraction, \( s \), of risky assets, \( a_0 \), it chooses to hedge by ceding risk to reinsurers given a reinsurance cost per unit of risk \( \phi \). Following Froot and O’Connell (2008), we assume those reinsurance contracts are linear, so that the hedged assets have a certain value of \( a_0s \). Without loss of generality, we set \( a_0 = 1 \) so \( a_0s = s \). At date \( t = 1 \), the insurer realizes an amount of the liquid asset \( a \) as it chooses to invest in a new project where\(^1\)

\[
a = s + (1 - s - s\phi)\bar{r}.
\]

The term \( (1 - s - s\phi)\bar{r} \) represents the realized payoff of the risky assets. The new project entails an initial investment \( I = a + e \). As such, in addition to the internal funds \( a \), the insurer must raise \( e \) from outside investors. The payoff \( G(I) \) of the project is realized at \( t = 2 \).

\(^1\)For an insurer, this project may involve the competitive pricing of insurance policies to expand or protect market share (Froot and O’Connell 2008).
To maximize $G(I)$ at date $t = 2$, the insurer optimizes its reinsurance network at date $t = -1$. We assume that each reinsurer is endowed with a risky portfolio $Y$ and the insurer evenly transfers risk to $n$ ($n \leq N$) reinsurers—that is, per unit ceded risk by the insurer, each reinsurer bears $X = \bar{r}/n$. In equilibrium, the insurer optimizes its reinsurance network with respect to outdegree centrality $n$ and outdegree constraint $C^o$, for which we will provide intuition and definitions later.

The setting of our model depicted above is similar to that of Froot et al. (1993), which explains the benefits of risk management for coordinating corporate investment and financing policies. As a departure from Froot et al. (1993), we explicitly consider the impact of the social and transaction cost factors on risk management decisions. As we will show in Section 3, the reinsurance cost $\phi$ is determined by the properties of the insurer’s network resources, the mechanisms of which have an important implication for the insurer’s reinsurance demand.

The above discussion is summarized in the timeline presented in Figure 1.

2.2. Reinsurance Network Theoretical Background

Cost is a major consideration in the process of risk management (Schmit and Roth 1990). While important, the notion of risk management costs has rarely been directly investigated. There is only limited empirical evidence indirectly showing the role of hedging cost in explaining a positive relationship between risk management level and firm size. For example, by investigating a large sample of over 3,000 firms, Mian (1996) finds that firm size is positively related to corporate hedging behaviors. Likewise, Cummings, Phillips, and Smith (2001) show that large insurance firms are more active in participating in the derivatives market. Haushalters (2000) study of 100 oil and gas producers and Graham and
Rogers (2002) investigation of 442 firms report similar results on the pattern of firm size-hedging linkage. Together these studies support the scale economies hypothesis that the high fixed costs of a risk management program enable large firms to benefit more from hedging activities as compared to their smaller counterparts. While these studies provide indirect evidence on the importance of risk management cost, they treat costs of risk management as exogenous and independent of the social context within which the firm is embedded. This unfortunately limits our understanding of corporate hedging behaviors, particularly in the insurance industry where reinsurance constitutes a popular risk management method. In fact, Powell and Sommer (2007) show that during the years 1996–1999, more than 90% of the U.S. property and casualty insurers cede risk through reinsurance. To manage risk assumed from insurers, reinsurers purchase and sell reinsurance polices with each other, creating risk transfer networks in reinsurance markets. Such reinsurance networks in turn can affect an insurer’s risk management cost as extensive fieldwork suggests networks are responsible for determining various costs. Yet, so far, the literature has not systematically examined how reinsurance networks relate to reinsurance costs. To fill out the gap, we consider how these costs are determined by reinsurance network structural patterns that influence the extent to which insurers participate in reinsurance over time. Specifically, we distinguish among three reinsurance costs in the network context: loadings, contagion costs, and search and monitoring costs.

Consider first the implications of networks for loadings. An insurer will generally pay loadings over and above the expected risk transferred. The loadings charged by reinsurers correspond to the costs of bankruptcy associated with the assumed risk (Jean-Baptiste and Santomero 2000). To the extent that the risks of those reinsurers are not perfectly correlated, as the literature on risk sharing suggests, other things equal, the total loading will be lower if the insurer partners with more reinsurers (i.e., a higher network centrality) than otherwise.

Contagion costs constitute a second important reinsurance cost in our setting. Recent turbulence in financial markets has called into question the belief of noncorrelation or low correlation of risks among risk takers. In September 2008, the financial giant American International Group (AIG) was teetering on the brink of bankruptcy caused by losses from heavy participation in credit default swaps. By selling swaps, AIG had guaranteed the performance of borrowers in the event of default. A wave of foreclosures of subprime mortgages hit AIG hard, which threatened to set off a massive domino effect around the world because its numerous business ties provided channels for the propagation of financial distress.
Similarly, in the reinsurance market, linkages among reinsurers (i.e., network cohesion) are a concern to an insurer given the possibility that a shock to one reinsurer may spread to others. This means that the insurer will need to consider network default risk in addition to individual insolvency risk. In short, the insurer with a more cohesive network among reinsurers will tend to have a higher contagion cost.

Third, reinsurance costs also include costs from uncertainties in determining appropriate (competitive) prices, search and information costs, and costs of monitoring and enforcing contractual performance (Williamson 1979, MacMinn 1980). Robins (1987, p. 69) argues that “although these costs are independent of the competitive market price of the goods or services, they are determined by the nature of the exchange.” Issues such as the difficulty of reaching cooperative agreement, enforcing contractual promises, or accessing needed information are instrumental in determining these costs. In this regard, an insurer’s interaction with more reinsurers will create higher search and monitoring costs as the complexity and the information requirement for coordination increase with network centrality. The cohesion of the network is also relevant. As will be discussed in greater detail below, insurers located in networks where reinsurers are closely connected may capitalize on their positions because a dense network promotes a normative environment, facilitates trust, breeds mutual understanding, and alleviates opportunistic behaviors, which lowers the need for deliberate information collection and sanctions. Following sociologists, we refer to the benefits tied to cohesive networks as “social capital” (Coleman 1988, Gulati 1995). In our model, the presence of cohesive ties among reinsurers may be part of an optimal network design. This implies linkages among reinsurers may be desirable to the insurer despite the presence of financial contagion. The incorporation of the notion of social capital has important implications for the finance and insurance literature in that it complements the theory of financial contagion (Allen and Gale 2000, Lagunoff and Schreft 2001) and offers a more balanced view of network cohesion.

2.3. Reinsurance Network Analysis

Network measures, outdegree centrality $n$ and outdegree constraint $C^o$, capture various aspects of a network’s economic role in determining reinsurance costs. We defer a detailed discussion of the cost implications of different network structures to Section 3. Here, we focus on their constructs and interpretations.
2.3.1. Outdegree Centrality

*Outdegree centrality* captures the number of reinsurers to which an insurer chooses to transfer risk. The more ties that exist, the more opportunities for risk exchange. We create a binary variable $y_{ij}$ that equals 1 if insurer $i$ cedes risk to reinsurer $j$ ($j = 1, 2, \ldots, N$), and zero otherwise. Insurer $i$ hedges risk when $\sum_j y_{ij} \geq 1$. That is, insurer $i$ will have at least one risk ceding relationship. If insurer $i$ decides to transfer risk to $n$ reinsurers, the *outdegree* of insurer $i$ equals $\sum_j y_{ij} = n$.

2.3.2. Outdegree Constraint

The insurer also considers its network cohesion, that is, the linkages among its reinsurers. Linkages among reinsurers are relevant to the insurer because they form channels for knowledge diffusion and financial contagion. Formally, given $i \neq j$ and $i \neq q$, we denote by $Y^o_{ij}$ the proportion of insurer $i$’s network associated with its risk transfer to reinsurer $j$,

$$Y^o_{ij} = \begin{cases} \frac{y_{ij}}{\sum_q y_{iq}} & \text{if } \sum_q y_{iq} > 0 \\ 0 & \text{if } \sum_q y_{iq} = 0 \end{cases}.$$  \hspace{1cm} (2)

As in Burt (1992), our evaluation of the role of reinsurer $j$ in insurer $i$’s network takes into account whether another reinsurer $q$ of insurer $i$ also cedes risk to $j$. Insurer $i$’s network would be cohesive when (i) insurer $i$ invests in a large proportion of network relationship with reinsurer $q$; and (ii) the proportional strength of $q$’s relationship with $j$ is strong (see Burt 1992). To illustrate this idea, we define:

$$Y^o_{iq}Y^o_{qj},$$  \hspace{1cm} (3)

where given $i \neq p$ and $i \neq q$,

$$Y^o_{iq} = \begin{cases} \frac{y_{iq}}{\sum_p y_{ip}} & \text{if } \sum_p y_{ip} > 0 \\ 0 & \text{if } \sum_p y_{ip} = 0 \end{cases},$$  \hspace{1cm} (4)

and, given $q \neq j$ and $q \neq l$,

$$Y_{qj} = \begin{cases} \frac{y_{qj} + y_{lj}}{\sum_l(y_{ql} + y_{lq})} & \text{if } \sum_l(y_{ql} + y_{lq}) > 0 \\ 0 & \text{if } \sum_l(y_{ql} + y_{lq}) = 0 \end{cases}.$$  \hspace{1cm} (5)
A high value of the product in Eq. (3) implies a dense reinsurance network between insurer \( i \) and reinsurer \( j \). Aggregating the product in Eq. (3) across all reinsurers \( q \) and adding insurer \( i \)'s direct connection with \( j \), the expression squared defines the network cohesion of insurer \( i \) with reinsurer \( j \),

\[
C_{ij}^o = \left( Y_{ij}^o + \sum_q Y_{iq}^o Y_{jq}^o \right)^2, \quad i \neq q \neq j.
\] (6)

Then, insurer \( i \)'s network cohesion, called outdegree constraint, equals the sum of \( C_{ij}^o \) across \( j \),

\[
C^o = C_i^o = \sum_j C_{ij}^o.
\] (7)

A higher \( C^o \) indicates a higher degree of interdependence among \( n \) reinsurers.

Using a numerical example, the appendix shows in detail how outdegree centrality \( n \) and outdegree constraint \( C^o \) are calculated.

3. Characterization of the Optimal Reinsurance Network

This section focuses on a typical reinsurance network: one in which some or all reinsurers are directly or indirectly linked to each other (as depicted in Figure 2).²

![Figure 2. Reinsurance Network](image)

Nodes in Figure 2 represent insurer \( i \) (called “\( F_i \)”) and its reinsurers (denoted as “\( RT \)”). Arrows represent risk transaction ties, pointing from the insurer to the reinsurer. Two-directional arrows indicate

²The analysis and conclusions in this paper also apply to a reinsurance network in which each reinsurer stands on its own (i.e., no linkages among reinsurers). In this unique case, the value of the insurer's network cohesion equals 0.
that both firms (e.g., reinsurers 1 and N) on the arrow assume and cede risks. In addition, Figure 2 illustrates whether or not the reinsurers in the insurer’s network are connected. Two reinsurers are connected if there is a path between them (but not through the insurer). For example, reinsurer s is disconnected from all other reinsurers while reinsurer o is directly connected to l and N and indirectly linked to other reinsurers via l and N. As will be clear, the pattern of these connections has reinsurance cost implications for insurer i as a cedent.

3.1. Reinsurance Cost of the Insurer

The total reinsurance cost \( \phi \) per unit risk of the insurer is a function of both network centrality and cohesion. It equals

\[
\phi = \phi_r + \phi_c + \phi_s. \tag{8}
\]

The first cost component \( \phi_r \) is the loadings. The second cost component \( \phi_c \) is the contagion cost caused by linkages among reinsurers, and the last cost component \( \phi_s \) is the search and monitoring cost mitigated by the network social capital.

3.1.1. Loadings

The existence of convex costs in financial distress suggests that reinsurers are risk averse and susceptible to the variability of their assumed risks (Jean-Baptiste and Santomero 2000, Jin and Jorion 2006, Mayers and Smith 1982, Smith and Stulz 1985). Accordingly, they charge loadings (or risk premia) above their expected payments. In accord with Jean-Baptiste and Santomero (2000), we assume all reinsurers have the same convex distress cost function \( cR^2/2 \). Specifically, the assumed risk \( R \) acts as a proxy for the probability of financial distress, and the constant \( c/2 \) is the distress cost factor with \( c > 0 \). That is, all else equal, a higher business risk is associated with a higher cost of financial distress.

As mentioned earlier, the insurer’s risky asset \( \tilde{r} \) is normally distributed with a mean of \( \mu = 1 \) and a variance of \( \sigma^2 \). Without loss of generality, we assume the correlation \( \rho_{XY} \) between each reinsurer’s existing risky portfolio \( Y \) and its share of the insurer’s risk \( X \) equals 0. Given \( E[X] = E[\tilde{r}/\sum_j y_{ij}] = 1/n \) and \( \rho_{XY} = 0 \), the expected marginal financial distress introduced by \( X \) for reinsurer \( j \) is

\[
MC_j = E\left[\frac{c}{2}(X + Y)^2\right] - E\left[\frac{c}{2}Y^2\right] = \frac{c}{2}E[X^2]
\]

\[
= \frac{c(1 + \sigma^2)}{2n^2}, \tag{9}
\]
when \( y_{ij} = 1 \) (\( j = 1, 2, ..., N \)).

Obviously, the sum of expected marginal financial distress costs \( \phi_r \) per unit risk across \( n \) selected reinsurers equals

\[
\phi_r = \sum_j y_{ij} MC_j = \frac{c(1 + \sigma^2)}{2n}.
\]

(10)

Given that reinsurers price the risk at the zero profit point, \( \phi_r \) will be finally paid by the insurer as loadings so that the reinsurers just break even. This constitutes one of the reinsurance cost components of the insurer.

The term \( c/2n \) in Eq. (10) illustrates the benefits of transferring risk to more than one reinsurer: the more the reinsurers share the risk (i.e., higher \( n \)), the lower the total loading paid by the insurer. This part alone, in particular, is based on the assumption of no risk sharing among \( n \) reinsurers. However, we recognize social factors resulting from the embeddedness of those reinsurers in a rich risk sharing context can be influential in altering the uncertainty perceived by the insurer as a cedent. This idea reflects the potential financial contagion among reinsurers.

3.1.2. Contagion Cost

Firms often create linkages among themselves to insure against idiosyncratic liquidity shocks (Allen and Gale 2000). Such linkages allow for mutual insurance as firms with liquidity surpluses provide liquidity for firms with liquidity shortages as long as there is enough liquidity in the system as a whole, so called “risk sharing”.

To illustrate this, recall that each reinsurer has a risky portfolio \( Z = Y + X \) associated with \( z \) units of risk at \( t = 0 \) where \( Y \) represents each reinsurer’s existing risky portfolio and \( X \) is its share of insurer’s risk. The portfolio \( Z \), which yields a return of one unit after one period, entails a loss payment \( d_1 \) per unit risk to a proportion of total contracts, \( x \), at \( t = 1 \).

The realized proportion \( x \) of contracts being paid \( d_1 \) at date \( t = 1 \) varies from one reinsurer to another. We assume it takes one of two likely values, a high value \( x_H \) with probability \( p_H \) and a low value \( x_L \) with probability \( p_L \). Risk sharing among reinsurers provides a solution to an excess demand for cash in the state \( x_H \) by reallocating liquidity.

However, Allen and Gale (2000) show that risk sharing only redistributes liquidity, but it does not create liquidity. As such, the optimal risk sharing among reinsurers is achieved only if there is no
aggregate uncertainty regarding the probability of the payment $d_1$ at date $t = 1$. Otherwise, a small payment shock can generate financial contagion.

Specifically, assume each reinsurer is able to issue securities at a fair value $b$ per unit risk in normal times but with a payment shock they are sold at a discount and generate only $r_b b$ at date $t = 1$ with $0 < r_b < 1$. This means that raising capital from the outside to produce liquidity is costly.

When $r_b$ is too small to meet the extra demand for liquidity, i.e.,

$$r_b b < x_H d_1 - Z/z,$$  \hspace{1cm} (11)

the reinsurer will go bankrupt. Suppose there exists a probability of $p_j^c$ that reinsurer $j$ will cause an economywide liquidity shock $\epsilon$, leading to the aggregate payment at date $t = 1$ greater than the system’s ability to supply liquidity. When this happens, suppose the upper bound of $j$’s liquidation value per unit of risk equals $\bar{q}_j^c$.

The linkages in a group can spread the shock to reinsurers which have indirect claims on reinsurer $j$. The term indirect claim means there exists a cash flow path from one reinsurer to another although they are not directly linked. To illustrate, in Figure 2, the path

$$o \rightarrow \ell \rightarrow j,$$  \hspace{1cm} (12)

represents the case in which $o$ transfers risk to $\ell$ and $\ell$ transfers risk to $j$. In our terms, we say reinsurer $\ell$ has a direct claim and reinsurer $o$ has an indirect claim on reinsurer $j$. If reinsurer $j$ is bankrupt, $\bar{q}_j^c < d_1$. This implies there will be an immediate spillover effect to reinsurer $\ell$ ($\ell \neq j$ and $\ell = 1, 2, \ldots, k_{j2}$) which has a direct claim on reinsurer $j$. If reinsurer $\ell$ fails because of reinsurer $j$’s bankruptcy, reinsurer $o$, the indirect claimer of $j$, will be adversely affected as well. Consequently, the direct and indirect linkages among reinsurers are a concern to the insurer because a shock to one of its reinsurers may spread to the others.

As noted before, reinsurer $j$ assumes a share of unit risk, $X = \tilde{r}/\sum_j y_{ij}$, from the insurer. At the time of reinsurer $j$’s bankruptcy, the insurer will suffer an immediate loss no less than $(d_1 - \bar{q}_j^c)/\sum_j y_{ij}$. Furthermore, the linkages among reinsurers may spread $j$’s shock to other reinsurers in the insurer’s network. To capture the domino effect of cohesive ties among reinsurers in spreading financial shocks, which imposes convex costs on the insurer, we model the insurer’s expected contagion cost as a quadratic
function of its network cohesion measure, outdegree constraint $C^o$:

$$\sum_j y_{ij} p_j^c \frac{d_1 - \bar{q}^c}{\sum_j y_{ij}} (u_1 C^o)^2,$$

where $u_1$ is a positive constant and $p_j^c$ is the probability of financial contagion caused by reinsurer $j$.

Assume each reinsurer has an equal probability $p_j^c = p^c (j = 1, 2, ..., N)$ of being the one with an excess demand for liquidity and these crisis events are independent. Let $q^c$ be the highest liquidation value at bankruptcy among $\sum_j y_{ij} = n$ reinsurers,

$$q^c = \max \left( y_{i1} \bar{q}^c_1, ..., y_{ij} \bar{q}^c_j, ..., y_{iN} \bar{q}^c_N \right).$$

Accordingly, the lower bound of expected contagion cost to the insurer is

$$\phi_c = p^c (d_1 - q^c) (u_1 C^o)^2.$$ 

Eq. (15) suggests that the expected contagion cost to the insurer is a convex function of the degree of linkages among reinsurers.

3.1.3. Search and Monitoring Cost

To enter a contract that addresses its risk management needs while minimizing the risk imposed by moral hazard concerns, the insurer must first be aware of the existence of potential reinsurers and, second, have access to information about the reliability of those reinsurers. An important source of such information is a linked network in which the insurer resides (Hochberg et al. 2007). In this regard, a cohesive network plays a positive role in that it expedites knowledge transfer. A dense network also encourages reinsurers to honor their obligations through common third parties (Gulati 1995). Common third parties serve as an incentive to enhance local reputation and as an effective deterrent to opportunism. Reinsurers in a cohesive network are more likely to fulfill their responsibilities because failure to conform to the terms of a contract may jeopardize the defector’s reputation as a trustworthy counterparty and diminish its ability to enjoy future benefits from a good reputation. In keeping with the network literature, we use the term “social capital” to designate the benefits of a cohesive network. Accordingly, the search and monitoring cost in a linked network is given by

$$\phi_s = \sum_j y_{ij} (w - u_2 C^o) = wn - u_2 C^o n,$$
where \( w > 0 \) represents the search and monitoring cost expenses per counterparty without considering the benefit of social capital. \( u_2 \) is a positive constant and \( u_2C^o \) acts as the proxy for social capital per reinsurer, which reduces \( \phi_s \).\(^3\) Eq. (16) indicates that the network cohesion \( C^o \) plays a positive role in reducing search and monitoring costs.

3.2. Optimal Reinsurance Network

At \( t = 0 \), the insurer makes its reinsurance decision given the reinsurance cost \( \phi \) per unit of risk. At this time, both the wealth \( a \) and the investment opportunity are random. Following Froot et al. (1993), we define the project payoff \( G(I) \) in a world of changing investment opportunities as

\[
G(I) = \varphi g(I),
\]

where \( \varphi = \rho(\tilde{r} - \mu) + 1 = \rho(\tilde{r} - 1) + 1 \). The variable \( \rho \) stands for the correlation between the project and the risk to be hedged. The function \( g(I) \) is the expected level of output from the insurer’s project, provided that it is independent of the cash flow from its asset \( a \) in place at \( t = 1 \).

We can find the optimal reinsurance ratio \( s \) by maximizing the expected net profits \( E[V(a)] \) with respect to \( s \):

\[
\max_s E[V(a)],
\]

where

\[
V(a) = \max_a \varphi g(I) - I - T(e).
\]

Here the cost function \( T(e) \) measures the deadweight costs to the insurer from external finance. Costs of external finance have been discussed in articles by Jensen and Meckling (1976), Myers (1977), Townsend (1979), Myers and Majluf (1984), MacKay and Moeller (2007), and many others. Following these papers, our deadweight costs \( T = T(e) \) are modeled as a convex function of external financing, that is, \( T_e \geq 0 \) and \( T_{ee} \geq 0 \). The expectation is taken with respect to the rate of return \( \tilde{r} \). Given \( V_a \geq 1 \) and \( V_{aa} < 0 \), the optimal \( s \) will satisfy

\[
E \left[ V_a \frac{da}{ds} \right] = 0.
\]

Solving this optimization problem yields the following result.

\(^3\)If the reinsurance network is an unlinked network, \( u_2C^o = 0 \).
Lemma 1. Given a reinsurance cost $\phi$, the insurer’s optimal reinsurance ratio equals:

$$s = \frac{1}{1 + \phi} \left( 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} \right),$$  \hspace{1cm} (20)$$

where the second derivative of $V(a)$ with respect to $a$ equals

$$V_{aa} = \varphi g II \frac{dI}{da}. \hspace{1cm} (21)$$

Reinsurance networks can influence the risk management decision by exerting an impact on the opportunity set and costs perceived by insurers. Because the reinsurance cost reduces the internal fund $a$ available for investment at date $t = 1$ and internal funding $a$ is cheaper than external capital $e$, minimizing $\phi$ is equivalent to maximizing $V(a)$. We can find an optimal reinsurance network by solving the problem

$$\min_{n,C^o} \phi$$  \hspace{1cm} (22)$$

subject to

$$1 \leq n \leq N, \quad \text{and} \quad 0 < C^o < w/u_2.$$ 

The second constraint, $0 < C^o < w/u_2$, ensures non-negative search and monitoring costs. The resulting optimal network structure and the reinsurance ratio are given by the following lemma.

Lemma 2. Suppose the insurer’s network has $n$ linked reinsurers. Given $1 \leq n^* \leq N$ and $0 < C^{o*} < w/u_2$, the optimal number of reinsurers in the network, $n^*$, the optimal network cohesion, $C^{o*}$, and the optimal reinsurance ratio, $s^*$, are

$$n^* = \frac{1}{3} \left( 2D_2 w + C_1 - \frac{B_2}{C_1} \right), \quad C^{o*} = \frac{u_2}{6D_2} \left( 2D_2 w + C_1 - \frac{B_2}{C_1} \right),$$

$$s^* = \left( 1 + \frac{R}{n^*} + n^* w + \frac{(n^*)^2}{4D_2} - \frac{u_2 (n^*)^2}{2D_2} \right)^{-1} \left( 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} \right),$$

where

$$R = c(1 + \sigma^2)/2, \quad D_2 = u_1^2 p^c(d_1 - q^c),$$

$$B_2 = -4D_2^2 w^2, \quad C_1 = \left( 8D_2^3 w^3 - 27D_2 R + (729D_2^2 R^2 - 432D_2^4 R w^3)^{1/2} \right)^{1/3}.$$
The term $\left(1 + \frac{R}{n^*} + n^*w + \frac{(n^*)^2}{2D_2} - \frac{u_2(n^*)^2}{2D_2}\right)^{-1}$ measures the effect of reinsurance cost on the optimal reinsurance ratio $s^*$. The reinsurance cost is determined by the tradeoff from changing the number of reinsurers $n$, captured by the term $\frac{R}{n^*} + n^*w$. The terms $\frac{(n^*)^2}{2D_2}$ and $-\frac{u_2(n^*)^2}{2D_2}$ in $s^*$ highlight the tradeoff between the cost of contagion present in linked networks and the benefit of social capital within cohesive networks, both of which are a function of network cohesion $C_o$.

3.3. Network Effect on Reinsurance

The above discussion characterizes the insurer’s optimal network. Network position determines an insurer’s reinsurance cost and, consequently, influences the insurer’s reinsurance decision. The following proposition demonstrates this.

**Proposition 1.** The reinsurance level $s$ of the insurer is a nonlinear function of the number of its reinsurers $n$: (i) $s$ is increasing in $n$ when $1 \leq n < \sqrt{R/(w - u_2C_o)}$; and (ii) $s$ is decreasing in $n$ when $n > \sqrt{R/(w - u_2C_o)}$.

Notice Proposition 1 illustrates the tradeoff from changing the number of reinsurers $n$. On the one hand, the loading $\phi_r$ is decreasing in $n$. On the other hand, the search cost $\phi_s$ is increasing in $n$. When $n$ is beyond $\sqrt{R/(w - u_2C_o)}$, the total marginal increase in $\phi_s$ exceeds the reduction in $\phi_r$. Accordingly, the reinsurance level $s$ is curvilinearly related to $n$.

Risk sharing is a common practice by reinsurers to diversify risk. However, if the reinsurers of the insurer mainly share risk with each other rather than with the other reinsurers out of the network, all else equal, the insurer will experience high contagion costs. Consequently, the reinsurance level of the insurer can be distorted by a high contagion cost caused by dense linkages among its reinsurers. On the other hand, given the benefit of social capital, cohesive linkages can lower search cost and alleviate some of risks of opportunism. Together, these arguments lead to Proposition 2.

**Proposition 2.** Given $n$ reinsurers that assume the insurer’s risk, all else equal, there exists an inverted U-shape relationship between the reinsurance level $s$ of the insurer and its network cohesion $C_o$: (i) $s$ is increasing in $C_o$ when $0 < C_o < u_1n/(2D_2)$; and (ii) $s$ is decreasing in $C_o$ when $C_o > u_1n/(2D_2)$.

Proposition 2 shows that the change in network cohesion affects the insurer’s reinsurance ratio through both the social capital and contagion cost. A cohesive network is beneficial when the positive effect
of social capital on accessing information and fostering norms is high and the contagion cost is low. However, beyond a certain point, the effect of financial contagion will turn the cohesive network into an obstacle for risk transfer. When \( C^o \) exceeds \( u_1 n/(2D_2) \), network cohesion is no longer advantageous and will make further reinsurance unattractive. Hence, the insurer will optimally cede less risk.

Our theoretical model is graphically depicted in Figures 3 and 4.

**Figure 3.** Effects of Network Centrality on Reinsurance Ceded

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**Figure 4.** Effects of Network Cohesion on Reinsurance Ceded
4. **Empirical Evidence**

In this section, we explore empirically the implications of our model for the reinsurance demand given an insurer’s reinsurance network resources. Specifically, we test (i) whether there exists a curvilinear (inverse U-shaped) relationship between the reinsurance level of an insurer and the number of its reinsurers and (ii) whether the reinsurance level of an insurer is curvilinearly correlated with the cohesion among its reinsurers.

4.1. **Data and Variable Definitions**

4.1.1. **Data**

The financial data and the data on reinsurance positions were taken from the annual regulatory statements filed by insurers with the National Association of Insurance Commissioners (NAIC) from 1993 to 2005. This data set is unique in the sense that we can identify virtually all reinsurers of reinsurance transactions, allowing the observation of each insurer’s entire reinsurance network in each year. The present sample consists of all U.S. property and casualty insurers for which data on a complete list of variables are available for testing our predictions. We exclude firms reporting zero or negative surplus, assets, premiums, losses or expenses and firms with reinsurance ratios above 100%. For the purpose of this study, we concentrate solely on primary insurers and examine how they transfer business risk given their reinsurance networks. Our sample encompasses both groups of insurers under common ownership and unaffiliated single insurance firms. For affiliated firms, data are aggregated by groups “because insurers formulate investment and risk management strategies at the overall corporate level” (Cummins et al. 2007). The resulting sample is an unbalanced panel containing 9,490 observations (1,262 distinct insurers) for the 13-year period. Our sample accounts for 96.4 percent of total industry premium volume in 2005.

4.1.2. **Reinsurance Variable**

To measure reinsurance demand, we use the level of reinsurance usage, which is defined as the percentage of reinsurance premium ceded over total premium written (Cole and McCullough 2006). It

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4 We exclude 6 observations with the reinsurance ratio above 100%. A reinsurance ratio above 100% means those firms cede more risk than they assume. This may occur when an insurer exits the insurance market.

5 The A.M.Best defines a primary insurer as any firm whose reinsurance assumed from nonaffiliates is less than 75 percent of the direct business written plus reinsurance assumed from affiliates (Cole et al. 2007).

6 Total premium written is the sum of direct premium written and reinsurance premium assumed.
represents the proportion of the underwriting risk that is laid off by primary insurers (i.e., insurers) to their reinsurance providers (i.e., reinsurers). If this ratio equals zero, the primary insurer retains 100% of its underwriting risk and cedes nothing. If this ratio is one, the primary insurer hedges fully and completely removes underwriting loss exposure from its books.

4.1.3. Independent Variables

Independent variables fall in two categories: variables to test our model and other control variables. Independent Variables to Test the Model. Variables suggested by our model consist mostly of various measures of network centrality and cohesion. Given that the term of reinsurance contracts is typically one year (Cummins et al. 2008a), we construct the network measures based on one-year windows.

Our network analysis of reinsurance views the network structural environment as providing opportunities for or constraints on risk transfer. These opportunities and constraints can be measured by how “central” the insurer is, based on the extent to which the insurer is involved in risk ceding relationships with reinsurers in the market. On the one hand, linkages with a wide range of reinsurers convey opportunities for risk exchange. As shown in Section 3, access to more reinsurers implies a lower total loading to the insurer. On the other hand, the insurer’s search and monitoring costs increase with the number of associated reinsurers. To capture these two opposing effects, in addition to outdegree introduced in Section 2, we use two other measures of centrality for our network analysis: degree and eigenvector.

Degree counts the number of unique ties with the insurer. This measure does not consider the direction of the ties. Thus, the insurer’s degree can be obtained by counting the number of lines incident with it. We let a binary variable $x_{ij}$ equal to 1 if at least one risk trading relationship exists between insurer $i$ and reinsurer $j$, and zero otherwise. Accordingly, the degree of insurer $i$ equals $\sum_j x_{ij}$. To ensure comparability over time, following Hochberg et al. (2007), in a network of size $n + 1$ with one insurer and $n$ reinsurers, we normalize outdegree and degree by the highest possible degree (i.e., $n$).

While degree centrality measure simply counts the number of reinsurers, eigenvector centrality takes into account the importance of each reinsurer in the insurer’s network. Formally, the eigenvector of insurer $i$ (denoted $ev_i$) is given by $ev_i = a \sum_j x_{ij} ev_j$, where $a$ is a parameter required to give the equation a non-trivial solution.\footnote{Specifically, $a$ is the reciprocal of an eigenvalue.} In essence, eigenvector assigns relative scores to all reinsurers in the network based on the principle that connections to high-centrality reinsurers contribute more to the centrality of...
the insurer than equal connections to low-centrality reinsurers. We use normalized eigenvector centrality, which is the eigenvector centrality divided by the maximum possible eigenvector centrality in a network of \( n + 1 \) firms.

Focusing on network centrality without considering cohesion (i.e., the level of inter-linkages among firms) can limit a network analysis in significant ways (Burt 1992). Essentially, balancing benefits and costs of network cohesion is a question of optimizing reinsurance decisions. Given the insurer \( i \)'s network, cohesive ties among reinsurers, on the one hand, tend to decrease search and monitoring costs, but on the other hand, increase contagion risks (see Proposition 2). Outdegree constraint \( (C_{i}^{o}) \) defined in Section 2 is a one-directional cohesion measure because it focuses only on risk transfer. But in some situations, firms share risk with each other and assume and cede risk at the same time. In this case, the network cohesion effect may spread to a greater extent. To incorporate this possibility, following Burt (1992), we define another measure, constraint \( (C_{i}) \), to captures the two-directional network cohesion of insurer \( i \):

\[
C_{i} = \sum_{j} C_{ij} = \sum_{j} \left( Y_{ij} + \sum_{q} Y_{iq} Y_{qj} \right)^2, \quad q \neq i \neq j, \tag{23}
\]

where the proportion of insurer \( i \)'s relation with reinsurer \( j \) in all \( i \)'s risk exchange contacts equals

\[
Y_{ij} = \frac{y_{ij} + y_{ji}}{\sum_{q} (y_{iq} + y_{qj})}, \quad q \neq i \neq j. \tag{24}
\]

The variables \( Y_{iq} \) and \( Y_{qj} \) are similarly defined.

In sum, the main network centrality variables we use are outdegree, degree and eigenvector centrality. The variables outdegree constraint and constraint measure network cohesion. To homogenize the network variables across firms and over time, we use normalized network centrality measures.

Other Independent Variables. Several control variables suggested by the existing literature are included in our empirical analysis. A long-term contracting relationship reduces information problems, leading to a lower reinsurance cost and a higher reinsurance level for the insurer. In order to investigate the duration of contract relationships and determine the impact of contract sustainability on reinsurance purchases, we construct a reinsurance sustainability index, following Garven and Grace (2007). This index is defined as the proportion of premiums ceded over a three-year period to reinsurance providers which are present in all these three years. Specifically, the value of the index for each primary insurer
in year $t$ is based upon the $[t - 3, t]$ window. It shows whether and to what extent primary insurers keep long-term relationships with the same reinsurers.

There are two theoretical frameworks that analyze the relationship between reinsurance and diversification. The first one, pioneered by Borch (1962), argues that a concentrated insurer in a given line of business or a geographic area can benefit from reinsurance because the use of reinsurance can increase diversification of risk (Cole and McCullough 2006). The second framework originates from the corporate risk management literature and emphasizes real services provided by the reinsurers. Mayers and Smith (1990) argue that the motivations of primary insurers to purchase reinsurance resemble those of any nonfinancial firms for insurance. Reinsurers have comparative advantages and expertise in risk management so they can provide real services to primary insurers. In particular, those services are more valuable to widely diversified firms since highly focused firms are more likely to accumulate the required knowledge in-house (Lucas et al. 2006, Parlour and Plantin 2008).

To test the impact of diversification on reinsurance usage, we use the Herfindahl index to measure business and geographic concentration. Line-of-business Herfindahl index is computed as the Herfindahl index of the percentage of premiums in each line of business written by an insurer; and Geographic Herfindahl index is computed as the Herfindahl index of the percentage of premiums written by an insurer in each state. A higher Herfindahl index indicates that the insurer is concentrated in fewer lines of business or in fewer states. The first framework (i.e. the diversification benefit argument) predicts a positive relationship between Herfindahl index of the primary insurance business and the use of reinsurance while the second (i.e. the real services hypothesis) suggests a negative relationship.

Prior studies suggest that cash flow volatility affects a firm’s decision to purchase insurance. This can occur for two reasons. First, a firm’s cash flow volatility determines its probability of encountering financial distress. Thus, the firm would be more likely to hedge the more volatile its income (Nance et al. 1993). Second, given the progressivity of corporate tax rates, a firm can minimize its expected tax liabilities through a reduction in earnings volatility (Smith and Stulz 1985). Thus, we use the overall volatility variable, $\sigma_{CF}$, to capture the effect of cash flow volatility on primary insurers’ incentives to purchase reinsurance (Myers and Read 2001). The variable $\sigma_{CF}$ is defined as:

$$\sigma_{CF} = \sqrt{\sigma_L^2 + \sigma_V^2 - 2\sigma_{LV}},$$

(25)
where $\sigma_L$ is the volatility of losses estimated from a time series of loss ratios of six property lines of business and seven liability lines of business, $\sigma_V$ is the volatility of assets based on six asset return series, and $\sigma_{LV}$ is the covariance of losses and assets. The estimation procedures of $\sigma_{CF}$ follow Cummins et al. (2008b).

We also control other firm characteristics with the following variables: i) size of the firm is measured by the logarithm of total assets; ii) performance of the firm is approximated as return on assets; iii) leverage is measured as the ratio of total liabilities over total assets on the balance sheet; iv) liability growth rate is proxied by the growth rate of losses incurred; v) effect of price regulation is measured by the percentage of premiums in price regulated lines (primarily personal auto and workers compensation); vi) firm age is calculated as the number of years since the firm was incorporated; vii) dummy variables indicate a firm’s organization forms, such as whether it is a group or an unaffiliated single company, and whether it is a stock or a mutual company; viii) percentages of short-tail and long-tail commercial and personal lines are included to control different risk levels across lines of business (Cole et al. 2011). See Table 1 for detailed variable definitions.

Table 2 characterizes our sample of primary insurers. The estimation data contain 9,490 observations representing 1,262 unique firms during the period 1993–2005. The average sample firm ceded to non-affiliated reinsurers 25.56% of total premiums written and assumed from non-affiliates. The reinsurance sustainability index has a mean of 0.62, which suggests that the average firm placed 62% of its reinsurance with the same reinsurers for three years.

The average primary insurer has normalized outdegree of 0.65% and normalized degree of 1.07%. This means that the average firm transfers risk to 0.65% of all reinsurance providers active in the market and has risk exchange relationships with a little over 1% of all other firms. Together with the fact

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8Specifically, the property lines of business include automobile physical damage, special property, fidelity and surety, accident and health, credit, and financial and mortgage guarantee. Liability lines include automobile liability, other (commercial) liability, medical malpractice, workers compensation, special liability, commercial multiple peril, and homeowners/farmowners. Classification of lines as property and liability is based on Schedule P of the NAIC regulatory annual statement.

9The rate of return series are as follows: (1) Equities - the total return on the Standard & Poor’s 500 Stock Index; (2) government bonds - the Lehman Brothers intermediate term total return; (3) corporate bonds - Moody’s corporate bond total return; (4) real estate - the National Association of Real Estate Investment Trusts (NAREIT) total return; (5) mortgages - the Merrill Lynch mortgage backed securities total return; and (6) cash and invested assets - the 30-day U.S. Treasury bill rate.
that more than half of all reinsurance is purchased from the same reinsurers over a three-year period, these low outdegree centrality scores suggest that the primary insurers repeatedly transfer risk to a small set of reinsurance providers. That is, that relationships are relatively exclusive and stable. Eigenvector centrality, incorporating the “quality” (i.e., centrality) of the actors to which a firm is connected, averages 2.46% of the theoretical maximum.

We construct two cohesion measures, outdegree constraint and constraint. Outdegree constraint is 0.55 and constraint is 0.46 on average across 9,490 observations. Recall from Proposition 2, there exists an inverted U-shape relationship between the reinsurance ratio and the network cohesion, which is proxied by outdegree constraint and constraint.

All three network centrality measures and two cohesion measures exhibit a fair degree of variation, suggesting that the reinsurance network resources vary substantially across different primary insurers. Thus, reinsurance cost could be quite unequally distributed in this network.

Table 2 shows that the average sample insurer has $1,355.16 million of assets, with a range from $0.21 million to $126.46 billion. Fifty-seven percent of the 1,262 insurers in the sample are unaffiliated single companies. Table 2 also reveals that the average firm faces a leverage ratio of 0.58 and the overall volatility of 0.20.

4.2. Empirical Models and Results

4.2.1. Reinsurance Level and Network Centrality

To shed light on the effect of network centrality on an insurer’s reinsurance level, we estimate the following two-way fixed effect model of reinsurance usage:

\[
\text{Reins}_{i,t} = \alpha_0 + \alpha_1 \times \text{Centrality}_{i,t-1} + \alpha_2 \times \text{Centrality}_{i,t-1}^2 + \sum \delta' \text{Z}_{i,t} + \nu_i + \eta_t + \varepsilon_{i,t}. \tag{26}
\]

Specifically, for firm \(i\) in year \(t\), the dependent variable \(\text{Reins}_{i,t}\) represents the reinsurance level, which is the ratio of reinsurance ceded to the sum of direct business written and reinsurance assumed (Cole and McCullough 2006). \(\text{Centrality}_{i,t-1}\) is the network centrality preceding the current year reinsurance purchase. \(\text{Z}_{i,t}\) is a vector of variables representing the firm-specific characteristics. Finally, \(\nu_i\) is the fixed effect for insurer \(i\) and \(\eta_t\) is the fixed effect for year \(t\).
Proposition 1 suggests there exists an inverse U-shaped relationship between network centrality and reinsurance level. We therefore include one centrality measure $Centrality_{i,t-1}$ and its square term $Centrality_{i,t-1}^2$ in regression (26). To ensure that our results are not simply driven by reverse causality, that is, a higher reinsurance level enables an insurer to shape its network position, we construct the network centrality measures one year before the reinsurance decision is made. If these variables can help explain the reinsurance level next year, this will indicate that networking affects the risk management decision.

The estimated regression (26) results are presented in Table 3. The standard errors of estimates are adjusted for heteroscedasticity. The coefficients of outdegree and its quadratic terms in column (1) suggest a significant inverse U-shaped curve, which supports Proposition 1. In columns (2) and (3), the two undirected centrality measures, degree and eigenvector and their quadratic terms have significant effects on the reinsurance level. The reinsurance level first increases with degree (or eigenvector) centrality but subsequently decreases, consistent with our theory that at some point, search cost exceeds cost reduction from ceding risk to more reinsurance providers. This echoes the findings in column (1) of the same table that the outdegree centrality is curvilinearly associated with the reinsurance level.

Overall, the main conclusions from empirical results on regression (26) confirm the predictions of our theoretical models on network centrality: there is an inverted U-shaped curvilinear relationship between an insurer’s reinsurance level and the number of its reinsurers. That is, the reinsurance level first increases but then decreases with the number of reinsurers.

4.2.2. Reinsurance Level and Network Cohesion

The results in the previous subsection are important because they are consistent with the role of loadings and search costs in reinsurance decisions in terms of network centrality described in Section 3. However, beyond these two determinants for reinsurance cost, the model presented in Section 3 also suggests that dense linkages among reinsurers, on the one hand, reduce search cost and alleviate opportunistic behavior. On the other hand, they spread contagion and increase reinsurance cost. Together, they imply a curvilinear relationship between a primary insurer’s reinsurance usage and its network cohesion. For a deeper analysis of the process by which network cohesion is incorporated into the reinsurance decision, we re-estimate the reinsurance level $Reins_{i,t}$ by including the estimated cohesion proxy among
reinsurance providers in each primary insurer’s network, $\text{Cohesion}_{i,t-1}$. Consistent with Eq. (15) and Eq. (16), the model is specified as follows:

$$
\text{Reins}_{i,t} = \alpha_0 + \gamma_1 \times \text{Cohesion}^2_{i,t-1} + \gamma_2 \times \text{Cohesion}_{i,t-1} \times \text{Centrality}_{i,t-1} \\
+ \alpha_1 \times \text{Centrality}_{i,t-1} + \alpha_2 \times \text{Centrality}^2_{i,t-1} + \sum \delta' Z_{i,t} + \nu_i + \eta_t + \varepsilon_{i,t}. 
$$

(27)

The specification in regression (27) is the same as regression (26), except that the social capital measure $\text{Cohesion}_{i,t-1} \times \text{Centrality}_{i,t-1}$ and the contagion risk measure $\text{Cohesion}^2_{i,t-1}$ have been added. We use the cohesion measures, *outdegree constraint* squared and *constraint* squared respectively, to proxy the domino effect of a cohesive network to spread financial contagion. Based on the hypothesis that a primary insurer purchases less reinsurance the higher the contagion risk, we expect a negative coefficient $\gamma_1$. On the other hand, social capital, proxied by $\text{Cohesion}_{i,t-1} \times \text{Centrality}_{i,t-1}$ as in Eq. (16), reduces the search and monitoring cost, implying a positive sign of $\gamma_2$.

Table 4 presents the results based on the heteroscedasticity-consistent estimators. The results support our hypothesis on network cohesion, which suggests the tradeoff between contagion cost and social capital. Columns (1) through (3) of Table 4 report the results on *outdegree constraint* squared with or without controlling network centrality.\(^{10}\) *Outdegree constraint* squared is a one-directional contagion cost measure. It measures the extent to which risk ceding among reinsurance providers spreads out financial contagion in a primary insurer’s reinsurance network. To illustrate, in column (1) evaluated at the mean *Outdegree constraint* level, a one-standard-deviation increase in *outdegree constraint* squared is associated with around 1.34 percentage points decrease in the reinsurance ratio, all else equal. Similarly, we also find evidence of a negative impact of the two-directional contagion risk measure *constraint* squared in columns (4) through (6) of Table 4.\(^{11}\) In summary, a primary insurer suffers from a reinsurance network with a high contagion risk.

The positive and significant coefficients of *constraint* $\times$ *outdegree* across various specifications in Table 4 suggest that the social capital embedded in cohesive networks increases the primary insurer’s

\(^{10}\)The effect of our network cohesion measures (*outdegree constraint* and *constraint*) remains robust to the cases using other network centrality measures and combinations as those shown in Table 3. To conserve space, the results are not reported in Table 4.

\(^{11}\)As we expected, the coefficient of *constraint* squared is less significant (significant at 10%) than that of *outdegree constraint* squared since *outdegree constraint* squared is a direct (or better) measure to capture financial contagion effect.
This result indicates that dense linkages among reinsurance providers facilitate information transmission, reduce search cost, and promote a normative environment that ensures trust between the primary insurer and its reinsurance providers. Thus, the primary insurer will optimally transfer more risks. In this market, it seems that the benefit of social capital often dominates the cost of contagion among reinsurers. Specifically, when regression (6) of Table 4 is evaluated at the mean constraint and outdegree levels, a one standard deviation increase in the cohesion measure, constraint, raises the reinsurance ratio by 4.74 percentage points \[= 22.555 \times 0.65 \times 0.36 - 2 \times 1.169 \times 0.46 \times 0.36 - 1.169 \times 0.36^2\], a rise of 18.8\% \[= 4.74/25.26\]. Furthermore, given a total number of 2,830 reinsurers in 2005 and regression (6) evaluated at the mean constraint and outdegree levels, the reinsurance ratio will increase from 25.56 to 26.05 \[= 100 \times (1/2,830) \times (22.555 \times 0.46 + 3.810 - 2 \times 0.279 \times 0.65 - 0.279/2,830)] percentage points, or by 2\% \[= 26.05/25.56 - 1\], if the insurer transfers risk to an additional reinsurer in 2005.

Besides those of the network variables of interest, the coefficients for the firm characteristics variables in all models of Tables 3 and 4 are generally consistent with the existing literature. Most of the firm characteristics variables are statistically significant and display the expected signs based on previous studies (Myers 1977, Smith and Stulz 1985, Mayers and Smith 1990, Nance et al. 1993, Cole and McCullough 2006, Lucas et al. 2006, Plantin 2006, Garven and Grace 2007, Parlour and Plantin 2008).

4.3. Robustness of Empirical Results

We construct several additional sets of models to test the robustness of our results.\(^\text{13}\)

4.3.1. Change-in-Variables Analysis

We investigate the robustness of previous results with respect to variable changes, as opposed to variable levels. The use of change-in-variables regressions can address endogeneity issues (Weber 2006). Moreover, change-in-variables regressions can mitigate correlated omitted variables concerns if such

\(^{12}\)Since many reinsurers not only transfer risk to other reinsurance providers but also assume risk from other insurers, both risk ceding and risk taking transactions shape the flow of information and promote trust in the network. They produce social capital. The two-directional cohesion measure constraint account for both risk exchange relationships. Accordingly, constraint \(\times\) outdegree is a better social capital measure than one-direction construct outdegree constraint \(\times\) outdegree although we obtain similar but somewhat weaker (as expected) results based on outdegree constraint \(\times\) outdegree.

\(^{13}\)To conserve space, we do not report the results. The results are available upon request.
variables are time-invariant in the level variables regressions. We estimate the following regression using 7,786 firm-year observations:

\[
\Delta \text{Reins}_{i,t} = \alpha_0 + \gamma_1 \times \Delta \text{Cohesion}_{i,t}^2 + \gamma_2 \times \Delta (\text{Cohesion}_{i,t} \times \text{Centrality}_{i,t}) \\
+ \alpha_1 \times \Delta \text{Centrality}_{i,t} + \alpha_2 \times \Delta \text{Centrality}_{i,t}^2 + \sum \delta' \Delta \text{Z}_{i,t} + \nu_i + \eta_t + \varepsilon_{i,t},
\]

where all variables are as defined previously. Taking first differences (\(\Delta\)) reduces our sample size from 9,490 to 7,786. We eliminate the change in firm age variable from the change-in-variables regression since it is the same for all firms in different years (\(\Delta \text{firm age}_{i,t} = 1\)) in our sample.

The results confirm our first prediction by finding a positive and significant coefficient for \(\Delta \text{Centrality}\) and a negative and significant coefficient for \(\Delta \text{Centrality}^2\); the results also confirm our second prediction by finding a positive and significant coefficient for \(\Delta (\text{Cohesion} \times \text{Centrality})\) and a negative and significant coefficient for \(\text{Cohesion}^2\). Overall, these change-in-variables results confirm the earlier findings based on the network level variables.

4.3.2. Modified Outdegree Measures

Previously, we define network centrality as the number of reinsurers in an insurer’s network. In particular, outdegree centrality equals \(\sum_j y_{ij}\) where \(y_{ij}\) is a binary variable that equals 1 if insurer \(i\) cedes risk to firm \(j\). Given the fact that our data were aggregated at the group level, we may under-estimate the relations among the insurer and reinsurers. For example, if one insurer transfers risk to one reinsurer that is part of group \(j\), the value of \(y_{ij}\) will be one. If another insurer cedes risk to three reinsurers that are all in the same group \(j\), \(y_{ij}\) will also take on a value of one. However, these two situations are not the same. As a robustness check, we add a dummy variable to the regressions. This dummy variable equals 1 if the insurer cedes risk to more than one affiliate in a group and 0 otherwise. Our results with this dummy variable are consistent with the findings in Tables 3 and 4.

To further investigate the robustness of our results, we modify the above outdegree centrality to capture the possibility of ceding risk to more than one affiliate in the same group, which we call Modified outdegree:

\[
\text{Modified outdegree} = \sum_j (n_{ij})^{1/2},
\]
where \( n_{ij} \) is the number of reinsurers in group \( j (j = 1, 2, ..., N) \) to which insurer \( i \) chooses to transfer risk. If insurer \( i \) cedes risk to more than one reinsurer in group \( j \), \( (n_{ij})^{1/2} > 1 \). The higher \( n_{ij} \), the higher impact of group \( j \) on Modified outdegree. In addition, this measure can capture the lower cost of reinsurance when an insurer operates with multiple reinsurers in a group compared with the situation when the insurer cedes risk to the same number of reinsurers that are completely unaffiliated.\(^{14}\) Then we replace outdegree with Modified outdegree and rerun the regressions. In results not reported here, we find that the coefficient of Modified outdegree is positive and significant and the coefficient of Modified outdegree squared is negative and significant, which echo the findings in Table 3. Similar to the interaction term of outdegree and constraint in Table 4, the interaction term of Modified outdegree and constraint is positive and significant, consistent with the social capital benefit argument.

To further confirm this, we conduct the robustness check with another modified outdegree centrality measure, called Modified outdegree:\(^{14}\)

\[
\text{Modified outdegree}' = \sum_j (n_{ij})^{1/3},
\]

(30)

where we take \( 1/3 \) power of \( n_{ij} \). Again, the results support Propositions 1 and 2 because of a positive and significant coefficient of Modified outdegree', a negative and significant coefficient of Modified outdegree' squared, and a positive and significant coefficient of constraint \( \times \) Modified outdegree'.

4.3.3. Impact of Derivative Usage

While the existing literature suggests insurers use derivatives to hedge financial risk (Cummins et al. 2001, Powell and Sommer 2007), derivatives have not been fully exploited by insurance companies (Lin et al. 2012). From 2000 to 2006, the average derivatives usage rate is merely 2.5% of total firms in the U.S. property and casualty insurance industry (Song and Cummins 2008). To control for the potential impact of derivative hedging volume on reinsurance purchase decision, as a robustness check, we include in the control variables the level of derivative usage. Following Song and Cummins (2008) and Lin et al. (2012), we define the level of derivative usage as an insurer’s notional amount of all derivative positions for hedging purpose held at year end, normalized by its total assets. NAIC starts to provide digitally recorded derivative trading data in 2000. So we rerun our regressions with the level of derivative usage.

\(^{14}\)For example, Insurer A has three reinsurance partners, B1, B2 and B3, which all belong to the same group B. Its Modified outdegree equals \( \sqrt{1 + 1 + 1} = \sqrt{3} \). On the other hand, Insurer A’ has reinsurance ties with three unaffiliated partners, C, D and E. The Modified outdegree of Insurer A’ is \( \sqrt{1} + \sqrt{1} + \sqrt{1} = 3 \), which reflects the higher reinsurance cost that firm A’ faces than that of firm A with the Modified outdegree of \( \sqrt{3} \).
usage as an additional variable for the period 2000–2005. Consistent with Cummins et al. (2001), we find the coefficient of derivative usage is positive and significant, providing some evidence that insurers view reinsurance and derivatives as complements. As for the results of network measures ($outdegree$, $outdegree$ squared, $outdegree \ constraint$ squared, $constraint$ squared, and $constraint \times \ outdegree$), they are all consistent with the results in Tables 3 and 4. In sum, our main results are robust to a more recent period after controlling for derivative hedging.

5. Conclusions

The current literature focuses on the benefits of reinsurance but abstracts from the issue of reinsurance costs embedded in this process. While conventional wisdom often simplifies the problem by assuming no or exogenous reinsurance costs, we present an equilibrium model to study the implications of an insurer’s network resources for its reinsurance costs, given that reinsurance markets are characterized by strong relationships and networks. We argue that, after controlling for known determinants of reinsurance decisions, an optimal reinsurance network creates value via minimizing various reinsurance costs: loadings, contagion costs, and search and monitoring costs. This is the first study, to the best of our knowledge, to examine the relation between an insurer’s reinsurance decision and reinsurance costs in the context of networking.

The model characterizes the optimal reinsurance level, first focusing on the tradeoff between loadings and search and monitoring costs in terms of reinsurance ties. In our model, the decrease in loadings from an increase in the number of reinsurers balances the expected higher search and monitoring costs. Thus, one main conclusion of the model is that the optimal reinsurance level is a curvilinear function of the number of reinsurers.

Another main result of the model relates to network cohesion. The financial contagion theory developed by Allen and Gale (2000) and Lagunoff and Schreft (2001) predicts potential contagion threat caused by the linkages among risk takers (e.g. reinsurers). This theory implies no or low linkages, but it cannot account for cohesive networks chosen by hedgers (e.g. insurers) in practice. In this paper we challenge the conventional wisdom with a fresh look at the role of network cohesion, and argue that an insurer’s network in which reinsurers are closely connected may be optimal despite the threat of contagion. A dense network produces social capital because it promotes a normative environment, reduces
search cost, alleviates opportunistic behaviors, and facilitates trust. Hence, the insurer will, in general, need to trade off the benefit of social capital and the cost of financial contagion. As a result, our model predicts a nonlinear relationship between the optimal reinsurance level and network cohesion.

Our empirical study shows that the evidence in the U.S. property and casualty insurance market is consistent with the predictions of the model. These results shed new light on the role of reinsurance costs in reinsurance decisions. Given the tradeoff between financial contagion and social capital, our findings have clear ramifications for insurers’ choice of closely linked or sparsely connected reinsurers. In addition, our analysis provides a deeper understanding of benefits and costs from increasing reinsurance ties. An inverted U-shaped relationship between reinsurance level and network centrality implies there is an optimal number of reinsurers for the insurer.

We believe this paper presents the starting point of a new way of thinking about corporate risk management in the social context. As such, it leaves some questions unanswered and in turn opens lines for further research. For example, notice that our theory is developed under the assumptions that all reinsurers have the same credit quality and equally share an insurer’s risk. This implies that reinsurance costs as a function of each reinsurer’s characteristics are left unexploited in the model. We would likely obtain richer results from the model in which an insurer’s reinsurance costs differ from one reinsurer to another. Second, although our theoretical model can also be applied to loan sale market networks, CDS market networks and others, our empirical results are specific to the insurance industry. Can these findings be applied to other industries and be generalized out of sample? Third, in this paper, we focus on reinsurance networks because reinsurance is the dominating risk transfer approach adopted by insurers. In this market, there are only a limited number of insurers implementing other risk transfer methods such as derivatives. However, firms in non-insurance industries may undertake several means of risk transfer. It would be interesting to investigate the interaction between different types of risk transfer networks. We leave these questions for future research.

References


Appendix

Network analysis example

Consider the reinsurance network of the Ohio Mutual Insurance Group as a risk cedent in 2005 (labelled “Focus Firm 282”) as shown in Figure 5.\textsuperscript{15} Nodes in the graph represent the Ohio Mutual Insurance Group and its reinsurers, and arrows represent risk transaction ties between them.\textsuperscript{16} The direction of each arrow represents the risk exchange relationship. Arrows point from the risk cedent to the risk taker. Two-directional arrows indicate that both firms on the arrow assume and cede risks. Outdegree measures the number of risk takers to which the cedent transfers the risk. Visually, the Ohio Mutual Insurance Group transfers risk to eight risk takers: firms 7, 17, 149, 189, 329, 400, 1324 and 1782. Denote the focus firm, the Ohio Mutual Insurance Group, as firm $i$, firm 7 as firm $a$, firm 17 as firm $b$, firm 149 as firm $c$, firm 189 as firm $d$, firm 329 as firm $e$, firm 400 as firm $f$, firm 1324 as firm $g$, and firm 1782 as firm $h$. So the focus firm’s outdegree equals $n = \sum_j y_{ij} = 8$ where $j = a, b, c, ..., h$.

The outdegree constraint $C^o$ of the focus firm $i$, equals

$$C^o = C^o_i = \sum_j \left( Y^o_{ij} + \sum_q Y^o_{iq} Y^o_{qj} \right)^2, \quad i \neq q \neq j. \quad (31)$$

\textsuperscript{15}The Ohio Mutual Insurance Group provides protection for auto, home, farm and business needs.
\textsuperscript{16}The risk takers of the Ohio Mutual Insurance Group are insurance companies or reinsurance companies. Specifically, Firm 7 is the American International Group; Firm 17 is the Berkshire Hathaway; Firm 149 is the Munich American Holding Corporation; Firm 189 is the Farmers Mutual Hail Insurance Company of Iowa; Firm 329 is the Arch Insurance Group; Firm 400 is the PartnerRe Group; Firm 1324 is the Toa Reinsurance Company of America; and Firm 1782 is the Ohio Mine Subsidence Insurance Fund, a custodial account overseen by the Treasurer of the State and comprised of funds from premiums paid by homeowners in the designated counties (see www.ohiodnr.com/geo/insurance/tabid/11704/Default.aspx).
For the Ohio Mutual Insurance Group,

\[ Y_{ia}^o = Y_{ib}^o = Y_{ic}^o = Y_{id}^o = Y_{ie}^o = Y_{if}^o = Y_{ig}^o = Y_{ih}^o = \frac{1}{8}, \]

\[
\sum_q Y_{iq}^o Y_{qa} = \frac{1}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{1}{5} + \frac{1}{8} \times \frac{2}{10} + \frac{1}{8} \times \frac{1}{6},
\]

\[
\sum_q Y_{iq}^o Y_{qb} = \frac{1}{8} \times \frac{1}{6} + \frac{1}{8} \times \frac{2}{10} + \frac{1}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{1}{5},
\]

\[
\sum_q Y_{iq}^o Y_{qc} = \frac{1}{8} \times \frac{1}{5} + \frac{1}{8} \times \frac{2}{10} + \frac{1}{8} \times \frac{1}{6} + \frac{1}{8} \times \frac{2}{8} + \frac{1}{8} \times \frac{1}{5},
\]

\[
\sum_q Y_{iq}^o Y_{qd} = \frac{1}{8} \times \frac{1}{10} + \frac{1}{8} \times \frac{1}{6} + \frac{1}{8} \times \frac{1}{10} + \frac{1}{8} \times \frac{1}{8},
\]

\[
\sum_q Y_{iq}^o Y_{qe} = \frac{1}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{2}{10} + \frac{1}{8} \times \frac{1}{6} + \frac{1}{8} \times \frac{2}{10} + \frac{1}{8} \times \frac{1}{8},
\]

\[
\sum_q Y_{iq}^o Y_{qf} = \frac{1}{8} \times \frac{1}{10} + \frac{1}{8} \times \frac{1}{6} + \frac{1}{8} \times \frac{1}{10} + \frac{1}{8} \times \frac{1}{5} + \frac{1}{8} \times \frac{1}{10},
\]

\[
\sum_q Y_{iq}^o Y_{qg} = \frac{1}{8} \times \frac{1}{8} + \frac{1}{8} \times \frac{1}{10} + \frac{1}{8} \times \frac{1}{10}, \text{ and } \sum_q Y_{iq}^o Y_{qh} = 0.
\]
Then, we can obtain $C^o = 0.403$ for the Ohio Mutual Insurance Group.

**Proof of Lemma 1**

We solve the model backwards to explore the impact of network resources on the optimal reinsurance decision. That is, we start with the insurer’s investment decision for the second period. Specifically, given internal funds $a$ at $t_1$, the insurer must determine its external financing needs $e (= I - a)$ to maximize net expected payoffs:

$$
V(e) = \max_I \varphi g(I) - I - T(e).
$$

(32)

Moving to $t_0$, the internal wealth $a$ is random. So we differentiate the expected net profits with respect to $a$ to solve the following maximization problem,

$$
V(a) = \max_a \varphi g(I) - I - T(e).
$$

(33)

With a little algebra, we obtain

$$
V_a \tilde{r} = \rho g I V_a + V_{aa} (1 - s - s\varphi).
$$

(34)

Since we incorporate randomness in investment opportunities, the insurer must determine $s$ to maximize expected profits at time 0:

$$
\max_s E[V(a)],
$$

(35)

where the expectation is taken with respect to the rate of return $\tilde{r}$. The optimal $s$ should satisfy

$$
E \left[ V_a \frac{da}{ds} \right] = 0,
$$

(36)

where

$$
V_a = (\varphi g_I - 1) \frac{dI}{da} - T_e \left( \frac{dI}{da} - 1 \right),
$$

(37)

is the first derivative of $V(a)$ with respect to $a$, and

$$
\frac{da}{ds} = 1 - \tilde{r} - \varphi \tilde{r}.
$$

(38)

Then, by applying Rubinstein (1976), we can obtain the optimal reinsurance ratio $s$ as follows:

$$
s = \frac{1}{1 + \varphi} \left( 1 + \frac{E[\rho g I V_a/\varphi g_I]}{E[V_{aa}]} \right),
$$

(39)

where $\varphi = \rho(\tilde{r} - 1) + 1$.

**Proof of Lemma 2**

To minimize $\phi$, we write the first-order condition of optimization problem (22) as $\phi'(n) = 0$ and $
\phi'(C^o) = 0$, which yields

$$
\phi'(n) = -\frac{R}{n^2} + w - u_2 C^o = 0
$$

(40)

$$
\phi'(C^o) = -u_2 n + 2D_2 C^o = 0.
$$

(41)

Eq. (41) implies

$$
C^o = \frac{u_2 n}{2D_2}.
$$

(42)

Replacing $C^o$ in Eq. (40) with the expression in Eq. (42), we obtain

$$
u_2^2 n^3 - 2D_2 w n^2 + 2D_2 R = 0.
$$

(43)
After solving cubic Eq. (43), the optimal number of reinsurers in a linked network equals

\[ n^* = \frac{1}{3} \left( 2D_2 w + C_1 - \frac{B_2}{C_1} \right), \]  

(44)

where

\[ R = c(1 + \sigma^2)/2, \quad D_2 = u_2^2 p^2(d_1 - q^2) \]  

(45)

\[ B_2 = -4D_2^2 w^2, \quad C_1 = \left( 8D_2^3 w^3 - 27D_2 R + (729D_2^2 R^2 - 432D_2^4 Rw^3)^{1/2} \right)^{1/3}. \]  

(46)

From Eq. (42), the optimal network cohesion equals

\[ C_o^* = \frac{u_2}{6D_2} \left( 2D_2 w + C_1 - \frac{B_2}{C_1} \right). \]  

(47)

The second-order condition of optimization problem (22) with respect to \( n \) and \( C_o \)

\[ \phi''(n) = \frac{2R}{n^3} > 0, \quad \phi''(C_o^*) = 2D_2 > 0. \]  

(48)

So \( n^* \) and \( C_o^* \) are strict minimum points. Thus, the optimal reinsurance ratio in a linked network equals

\[ s^* = \left( 1 + \frac{R}{n^*} + n^* w + \frac{(n^*)^2}{4D_2} - \frac{u_2 (n^*)^2}{2D_2} \right)^{-1} \left( 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} \right), \]  

(49)

where

\[ n^* = \frac{1}{3} \left( 2D_2 w + C_1 - \frac{B_2}{C_1} \right). \]  

(50)

**Proof of Proposition 1**

To examine the relation between the reinsurance level \( s \) of the insurer and the number of its reinsurers \( n \) in a network, we solve

\[ \frac{\Delta s}{\Delta n} = \frac{\Delta s}{\Delta \phi} \cdot \frac{\Delta \phi}{\Delta n} = - \left( \frac{1}{1 + \phi} - \frac{1}{1 + \phi + \Delta \phi} \right) \left( 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} \right) \frac{\Delta \phi}{\Delta n}. \]  

(51)

Since the insurer is a cedent,

\[ 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} > 0. \]

Hence,

\[ \frac{\Delta s}{\Delta n} \begin{cases} > 0 & \text{if } 1 \leq n < \sqrt{R/(w - u_2 C_o)} \\ < 0 & \text{if } n > \sqrt{R/(w - u_2 C_o)} \end{cases}. \]

**Proof of Proposition 2**

To study the relationship between the reinsurance level \( s \) of the insurer and the network cohesion among its \( n \) reinsurers, we examine

\[ \frac{\partial s}{\partial C_o} = \frac{\partial s}{\partial \phi} \cdot \frac{\partial \phi}{\partial C_o} = - \frac{1}{(1 + \phi)^2} \left( 1 + \frac{E[\rho g I V_{aa}/\varphi g II]}{E[V_{aa}]} \right) \frac{\partial \phi}{\partial C_o}. \]  

(52)

It is obvious that

\[ \frac{\partial s}{\partial C_o} \begin{cases} > 0 & \text{if } 0 < C_o < u_1 n/(2D_2) \\ < 0 & \text{if } C_o > u_1 n/(2D_2) \end{cases}. \]

This means that given \( n \), the reinsurance level \( s \) first increases and then decreases with the network cohesion \( C_o \).
<table>
<thead>
<tr>
<th>Reinsurance variable (%)</th>
<th>Definitions</th>
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</thead>
<tbody>
<tr>
<td>Reinsurance ceded</td>
<td>Percentage of reinsurance premium ceded over the sum of direct premium written and reinsurance premium assumed</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Network centrality measures (%)</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdegree</td>
<td>Normalized number of unique reinsurers to which an insurer transfers risk</td>
</tr>
<tr>
<td>Modified Outdegree</td>
<td>Normalized number of unique reinsurers to which an insurer transfers risk considering square root of the number of affiliates in each unique reinsurer</td>
</tr>
<tr>
<td>Modified Outdegree’</td>
<td>Normalized number of unique reinsurers to which an insurer transfers risk considering 1/3 power of the number of affiliates in each unique reinsurer</td>
</tr>
<tr>
<td>Degree</td>
<td>Normalized number of unique reinsurers an insurer has risk exchange relationships (regardless of taking or ceding risk)</td>
</tr>
<tr>
<td>Eigenvector</td>
<td>Normalized eigenvector centrality that takes into account both the number of reinsurers and the importance of each reinsurer in an insurer’s network</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Network cohesion measures</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint</td>
<td>Cohesion of two-directional linkages among an insurer’s reinsurers</td>
</tr>
<tr>
<td>Outdegree constraint</td>
<td>Cohesion of risk transfer (i.e. one-directional) linkages among an insurer’s reinsurers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Firm characteristics</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinsurance sustainability index</td>
<td>Proportion of premiums ceded over a three-year period to reinsurance providers which are present in all these three years</td>
</tr>
<tr>
<td>Total assets ($m)</td>
<td>Total assets in million dollars</td>
</tr>
<tr>
<td>Return on assets</td>
<td>Net income/total assets</td>
</tr>
<tr>
<td>Leverage</td>
<td>Total liabilities/total assets</td>
</tr>
<tr>
<td>Geographic Herfindahl index</td>
<td>Herfindhal index calculated based on direct premium written from each state in which an insurer is licensed to do business</td>
</tr>
<tr>
<td>Line-of-business Herfindahl index</td>
<td>Herfindhal index calculated based on direct premium written from each line of an insurer’s business</td>
</tr>
<tr>
<td>Liability growth rate</td>
<td>Average 5-year growth rate of the total industry losses incurred for each line of insurance weighted by the proportion of the net premiums written by an insurer in each line of insurance.</td>
</tr>
<tr>
<td>Firm’s overall volatility</td>
<td>Overall volatility based on the volatility of losses, the volatility of assets and the covariance of losses and assets</td>
</tr>
<tr>
<td>Percentage of premiums in price regulated lines</td>
<td>Percentage of premiums written in personal auto insurance and workers compensation insurance</td>
</tr>
<tr>
<td>Firm age</td>
<td>Number of years since the firm was incorporated</td>
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<tr>
<td>=1 if single firm</td>
<td>Single firm indicator</td>
</tr>
<tr>
<td>=1 if mutual company</td>
<td>Mutual company indicator</td>
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<tr>
<td>=1 if stock company</td>
<td>Stock company indicator</td>
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<tr>
<td>Percentage in long-tail personal lines</td>
<td>Percentage of premiums in long-tail personal lines</td>
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<td>Percentage in short-tail personal lines</td>
<td>Percentage of premiums in short-tail personal lines</td>
</tr>
<tr>
<td>Percentage in short-tail commercial lines</td>
<td>Percentage of premiums in short-tail commercial lines</td>
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Table 2
Descriptive Statistics

<table>
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<tr>
<th>Reinsurance variable (%)</th>
<th>No.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinsurance ceded</td>
<td>9,490</td>
<td>25.56</td>
<td>23.05</td>
<td>0.00</td>
<td>18.21</td>
<td>99.95</td>
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<table>
<thead>
<tr>
<th>Network centrality measures (%)</th>
<th>No.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outdegree</td>
<td>9,490</td>
<td>0.65</td>
<td>1.07</td>
<td>0.03</td>
<td>0.27</td>
<td>12.99</td>
</tr>
<tr>
<td>Modified Outdegree</td>
<td>9,490</td>
<td>0.80</td>
<td>1.63</td>
<td>0.03</td>
<td>0.27</td>
<td>20.56</td>
</tr>
<tr>
<td>Modified Outdegree’</td>
<td>9,490</td>
<td>0.77</td>
<td>1.49</td>
<td>0.03</td>
<td>0.27</td>
<td>18.08</td>
</tr>
<tr>
<td>Degree</td>
<td>9,490</td>
<td>1.07</td>
<td>2.43</td>
<td>0.03</td>
<td>0.30</td>
<td>28.90</td>
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<tr>
<td>Eigenvector</td>
<td>9,490</td>
<td>2.46</td>
<td>3.48</td>
<td>0.00</td>
<td>1.14</td>
<td>26.73</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Network cohesion measures</th>
<th>No.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint</td>
<td>9,490</td>
<td>0.46</td>
<td>0.36</td>
<td>0.01</td>
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Table 3
The Effect of Firm Network Centrality on Reinsurance Decision

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Note: t-statistics based on heteroscedasticity-consistent (HCC) standard errors are reported in parentheses. *** and ** denote significance at the 1%, 5%, and 10% level (two-sided), respectively.
## Table 4
The Effect of Firm Network Cohesion on Reinsurance Decision

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<td>(1.79)</td>
<td>(1.74)</td>
<td>(1.78)</td>
<td>(1.66)</td>
<td>(1.63)</td>
<td>(1.71)</td>
</tr>
<tr>
<td><strong>Diagnostics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>69.98%</td>
<td>70.03%</td>
<td>70.11%</td>
<td>69.95%</td>
<td>70.02%</td>
<td>70.10%</td>
</tr>
<tr>
<td>Test - $H_0$: no fixed effects (F-value)</td>
<td>12.4 ***</td>
<td>12.1 ***</td>
<td>11.9 ***</td>
<td>12.4 ***</td>
<td>12.1 ***</td>
<td>11.9 ***</td>
</tr>
<tr>
<td>Test - $H_0$: cohesion = 0 and cohesion $\times$ centrality = 0 if included (F-value)</td>
<td>18.1 ***</td>
<td>17.2 ***</td>
<td>7.1 ***</td>
<td>10.5 ***</td>
<td>14.5 ***</td>
<td>5.4 ***</td>
</tr>
<tr>
<td>No. of observations</td>
<td>9,490</td>
<td>9,490</td>
<td>9,490</td>
<td>9,490</td>
<td>9,490</td>
<td>9,490</td>
</tr>
</tbody>
</table>

Note: $t$-statistics based on heteroscedasticity-consistent (HCC) standard errors are reported in parentheses.

***, **, and * denote significance at the 1%, 5%, and 10% level (two-sided), respectively.