Equilibrium and Welfare in Insurance Markets with Time-Inconsistent Consumers

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Abstract

This paper studies the implications of unobservable consumer naivete on selection and welfare in a competitive insurance market with moral hazard. We identify two distinct forces of sophistication driving the formation of market equilibriums. One is that sophistication helps individuals internalize the incentive to take high effort by adopting some commitment devices, and hence sophisticates are willing to pay a lower premium to increase insurance coverage than naifs. The other is that sophistication helps prevent the optimistic underestimation of loss probability, and thus sophisticates are willing to pay a higher premium to increase insurance coverage than naifs. We show that the above forces of sophistication can produce adverse selection, advantageous selection and the absence of risk-coverage correlation observed in various insurance markets. We also show that sophisticates can adopt some commitment devices to alleviate the externality caused by hidden naivete, and certain incentive insurance clauses such as rebate check are welfare-improving when advantageous selection appears.

Keywords: time-inconsistent preference, moral hazard, consumer naivete, selection, market solutions.

1 Introduction

Empirical studies in economics and behavioral ecology suggest that animals and humans exhibit time-inconsistent preferences with a “present-bias”: when evaluating trade-offs between two future events, as the dates of the events become closer, they appear to assign a higher relative weight to the event that takes place earlier. An individual with time-inconsistent preference often encounters the self-control problem. Based on their awareness of this problem,
i.e., the degree of consumer naivete, people are divided into sophisticates and naifs.² Sophis-
ticates foresee their self-control problem and incorporate it in the decision process, whereas
naifs do not foresee this problem and make decisions in a myopic way.

In recent decades, much attention is devoted to studying individuals’ self-control problem
and exploring its economic implications. Among others, Thaler and Shefflin (1981) propose
an agency model with a farsighted planner and a myopic doer to characterize this problem
and discuss its effect on individual saving behavior. Laibson (1997) analyzes the decisions
of sophisticates with access to an imperfect commitment technology (an illiquid asset) and
suggests that financial market innovation may reduce welfare by providing “too much” liq-
uidity. O’Donoghue and Rabin (1999) point out naifs procrastinate immediate-cost activities
and preproperate immediate-reward activities, while sophisticates mitigate procrastination
framework to study the agents susceptible to temptation in infinite horizon consumption un-
der uncertainty. DellaVigna and Malmendier (2004) introduce consumer naivete to optimal
contract design.

However, to the best of our knowledge, there is little research on the effect of the un-
observable consumer naivete on market equilibrium and welfare. One notable exception is
Eliaz and Spiegler (2006).³ Similar to Eliaz and Spiegler (2006), the primary interest of our
paper is to screen the degree of consumer naivete in the time inconsistent behavior concerning
precautionary activities. However, our paper differs from their work in two aspects. First,
Eliaz and Spiegler (2006) characterize the menu of contracts offered by a monopolistic firm
to screen agents’ degree of naivete, but we consider a competitive market. Second, our anal-
ysis lies in the moral hazard framework, in which the payoff structure depends endogenously
on individuals’ choices of actions, whereas Eliaz and Spiegler (2006) assume that the payoff
structure is exogenous. Several recent papers including Newhouse (2006) and Yan (2010) also
study the moral hazard problem under time-inconsistent preferences, but our paper differs
from theirs by considering both adverse selection and moral hazard. Spiegler (2011) provides
a nice overview of currently available models looking into consumer behavior and market
equilibrium within the time-inconsistent framework, with insurance markets as an important
example and application.

Our work is also motivated by practice. The evidence that people often encounter self-
control problems regarding the self-protection activity is abundant and growing.⁴ In health

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²Field evidence on 401(k) investment (Madrian and Shea 2001) and health club attendance (DellaVigna
and Malmendier 2006) show that there is some degree of naivete in the way people anticipate their future
choices. See also Bryan et al. 2010 for further examples.

³Heidhues and K˝ oszegi (2010) also address unobservable consumer naivete in the context of credit market.

⁴Intuition suggests that time inconsistency and biased beliefs play an important role in people’s decisions
over insurance plans (which refers to ones’ long-run preferences) and preventive care measures (which refers to
insurance market, Gallup reports that 74% of smokers would like to give up smoking, but only 4% to 7% of the attempts are successful; 59% of Americans said they wanted to lose weight, but at least 15% of them were still trying 7 years later (Bryan et al. 2010). Brockett and Golden (2007) suggest that impulsive behavior (lack of self control) is present in driving and financial risk taking, and use that to explain the robust positive statistical relationship between credit scores and automobile insurance losses. Empirical evidences have also documented different levels of naivete when individuals take on tasks involving delayed benefits. These include evidence on homework completion (Ariely and Wertenbroch 2002), on health club attendance (DellaVigna and Malmendier 2006), and on examination preparation (Wong 2008). There are also indirect evidences found in the insurance products newly available in the marketplace. For example, in the automobile insurance market, the rebate checks used in the Allstate Insurance Company’s “Safe Driving Bonus” program\(^5\) and the recent inventions of “usage-based insurance,”\(^6\) can be understood as external commitment devices to alleviate the self-control problem and screen consumers with different degrees of naivete.

Our exploration of the heterogeneity in consumer naivete in this paper produces rich results and insights in explaining various risk-coverage correlations observed in the marketplace.\(^7\) The main implications lie in the following four aspects. First, we identify two distinct forces of sophistication driving the formation of market equilibrium. Intuitively, an individual with time-inconsistent preference encounters self-control problem, which undermines her ability in conducting precautionary activity in two aspects: the individual is tempted to take low precautionary effort when she has a choice of taking high effort or not; furthermore she is induced to procrastinate taking high effort when she has the choice of when to take high effort. Sophistication plays a prominent role in overcoming the above self-control problems, ones’ short-run preferences). In fact, since precautionary activity is a typical task which involves immediate cost but delayed reward, people tend to procrastinate and hold incorrect beliefs (O’Donoghue and Rabin 1999). \(^5\)See http://www.allstate.com/auto-insurance/auto-insurance-features.aspx. In this program, for every six months of accident-free driving, the driver can earn a Safe Driving Bonus check for up to 5% of the premium. \(^6\)Insurance companies have introduced “usage-based” insurance products in the marketplace. For example, Progressive Insurance Company’s “snapshot discount” stipulates that the insured installs a tracking device in her vehicle for a period of time in order to get a discounted insurance quote. See http://www.progressive.com/auto/snapshot-discount.aspx. In addition, vehicle maintenance record has been shown to be predictive of automobile insurance losses. See Bair et al. (2012). \(^7\)An alternative is to assume that some individuals are time-consistent, while others are not. This alternative assumption may be of interest in some situations, such as the study of group behaviors in Makarov (2011). However, this alternative assumption yields very limited results in our framework. There are two cases for this alternative assumption. Firstly, time-inconsistent consumers are naive. In this case, time-consistent consumers and naifs cannot be screened ex ante since naifs believe they are time-consistent. Secondly, time-inconsistent consumers are sophisticated. Sophisticates are less likely to take high effort than time-consistent consumers due to their self-control problem. Therefore, when the cost of the precautionary action is low, there is a pooling equilibrium in which both time-consistent consumers and sophisticates prefer to take high effort and purchase the same partial insurance at low premium rate. When the cost becomes sufficiently high, there is another pooling equilibrium in which both time-consistent consumers and sophisticates take low effort and buy full insurance at high premium rate. Consequently this alternative assumption cannot produce rich explanations for the various empirically observed risk-coverage correlations in the market place. A formal analysis is available upon request.
leading to different types of market equilibriums. One is that sophistication leads the individual to invoke some commitment device to internalize the incentive for high effort. Thus sophisticates are willing to pay a lower premium to increase insurance coverage than naifs, leading to adverse selection. The other is that sophistication prevents the optimistic underestimation of loss probability, and in this case sophisticates are willing to pay a higher premium to increase insurance coverage and take higher effort than naifs, resulting in the opposite of adverse selection, or the so-called “advantageous selection.”

This finding is closely related to the mainstream literature on selection in insurance markets. The classic adverse selection model by Rothschild and Stiglitz (1976) predicts a positive correlation between risk occurrence and insurance coverage. Such a prediction is supported by empirical evidences in the markets for acute care health insurance and annuities (cf., Mitchell et al. 1999 and Finkelstein and Porteba 2004). However, advantageous selection is observed in the markets for life insurance, long-term care and Medigap insurance (cf., Cawley and Philipson 1999, Finkelstein and McGarry 2006 and Fang et al. 2008). Different mechanisms have been proposed to explain the seemingly contradictory evidences. de Meza and Webb (2001) argue that advantageous selection can be explained by hidden heterogeneity in individuals’ risk aversions. Unfortunately, such an explanation is empirically rejected by Cohen and Einav (2007) and Fang et al. (2008). Sonnenholzner and Wambach (2009) suggest hidden heterogeneity in patience as another possible reason. Sandroni and Squintani (2007, 2009) study the implications of overconfidence in the framework of Rothschild and Stiglitz (1976) and find that adverse selection arises when the fraction of overconfident individuals is sufficiently low, but advantageous selection occurs when the fraction of overconfident individuals is high. Spinnnewijn (2010) finds that heterogeneity in risk perception can produce both adverse selection and advantageous selection. Different from the above mentioned models, in our paper, the heterogeneity arises endogenously from moral hazard instead of an assumed exogenous risk perception.

Second, we show that there exists an equilibrium in which sophisticates take the same effort (and hence represent the same risk) as naifs but purchase more insurance than naifs, which predicts the absence of risk-coverage correlation. Chiappori and Salanié (2000) and Saito (2006) test the correlation between risk occurrence and insurance coverage in auto insurance markets and find evidence for the absence of risk-coverage correlation. Cohen and Siegelman (2010) discuss various reasons why a risk-coverage correlation may not be found in some pools of insurance policies. To our best knowledge, our paper is the first to present a formal explanation for this phenomenon. Combining this result with the first set of findings, Sandroni and Squintani (2009) find that different fractions of overconfident individuals in the market can lead to either adverse selection or advantageous selection. Therefore they suggest that their results may account for the lack of statistically significant risk-coverage correlation observed empirically. However, they were not able to show directly the existence of such an equilibrium. Conceptually, a possible intermediate case between adverse selection and advantageous selection is a pooling equilibrium in which the two types of

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we contribute to the literature on selection in insurance markets by showing that heterogeneity in the degree of consumer naivete can result in adverse selection, advantageous selection, and even the absence of risk-coverage correlation all observed in various insurance markets.

Third, departing from traditional wisdom (Rothschild and Stiglitz 1976, or “RS”), we show that under adverse selection, both sophisticates (low-risk individuals) and naifs (high-risk individuals) achieve their first-best choices, offsetting the externality caused by high-risk individuals and making market intervention redundant. In the RS equilibrium, low-risk individuals prefer to purchase full insurance but are forced to purchase partial insurance due to the existence of high-risk individuals. However, in our model sophisticates prefer to insure less in order to maintain enough incentive to exert high precautionary effort and become low-risk. This feature makes the first-best choice of sophisticates (low-risk individuals) incentive compatible with the first-best choice of naifs (high-risk individuals), thereby eliminating the externality caused by the naive (high-risk) individuals.

Fourth, we show that in advantageous selection, sophisticates prefer to purchase more insurance than naifs because unlike naifs they do not optimistically underestimate the loss probability. However, the externality caused by hidden naifs prevents insurance companies from offering first-best contracts for sophisticates. In this case, we show that certain incentive clauses currently used in insurance contracts such as a rebate check can help screen sophisticates and naifs, thus improving all individuals’ welfare.

Our model setup has the following characteristics. First, our dynamic decision model captures the time lag between insurance purchase and the self-protection activity. It easily accommodates the discrepancy between an individual’s long-run preference when purchasing insurance and her short-run preference when taking the precautionary activity. For tractability, we employ quasi-hyperbolic discounting to model time-inconsistent preference (Strotz 1956 and Laibson 1997).

Second, consumers are assumed to be homogeneous in their risk preference and self-protection ability. They have the same initial wealth and face the same potential loss. Therefore, the only heterogeneity among consumers is their degree of naivete in recognizing the self-control problem. For simplicity, we assume that there are only two types of consumers, one is the naive, fully unaware of the self-control problem, and the other is the sophisticated, fully aware of it.9

Sophistication is manifested by invoking some commitment devices to control the short-individuals are of the same risk and purchase the same coverage, which is not equivalent to the absence of risk-coverage correlation.

9Since our qualitative results only rely on the assumption that some consumers are more naive than others, introducing partial naivete into the analysis does not produce more insights.
run selves. We formalize the intuition that time-inconsistency undermines an individual’s ability for precautionary activities in two different but related ways: it induces the individual to take low effort when she faces the choice of taking high effort or not; it also further induces the individual to procrastinate taking high effort when she faces the choice of when to take high effort. Accordingly, two functions of commitment devices are explored: one is to provide the incentive to exert high precautionary effort by altering the individual’s payoff structure; and the other is to mitigate procrastination and facilitate an objective perception of loss probability. To this end, we select two prototype commitment devices— incentive lottery and membership— to facilitate our analysis. While a versatile collection of commitment devices are used in practice, they all exhibit the above two functions to mitigate the individual’s self-control problem.\textsuperscript{10} In this paper, we focus on these functions of the commitment devices, not their specific forms.

Third, we follow the literature (O’Donoghue and Rabin 2001, DellaVigna and Malmendier 2004, and Gruber and K˝oszegi 2004) to use the individual’s long-run preference as the welfare measure. The main reason is as follows: ex ante, all earlier selves of the individual evaluate the benefit and cost based on the long-run preference; ex post, the individual would be grateful if she were forced to take action as her long-run self wishes.

The layout of this paper is as follows. Section 2 describes sophisticates’ and naifs’ preference for insurance contracts and precautionary activities. Section 3 characterizes the first-best contracts when insurance companies have perfect information regarding a consumer’s naiveté. In Section 4, we study the formation of adverse selection, the pooling equilibrium, and an equilibrium where there is a lack of risk-coverage correlation. In Section 5, we extend the baseline model to allow procrastination. Section 6 shows how advantageous selection can appear when procrastination is possible. We conclude the paper in Section 7.

2 Time-inconsistent individual’s preference

We develop a two-period model to characterize an individual’s dynamic insurance decision process: she first purchases an insurance contract and later employs the self-protection activity. Following the literature (e.g., de Meza and Webb 2001 and Sonnenholzner and Wambach 2009), we assume there are two effort levels, each corresponding to a different accident-prevention technique. The timeline of the individual’s decision process is given in Figure 1.

\textsuperscript{10}The commitment devices can be external commitments or internal commitments. External commitments involve real economic penalties (rewards) for failure (success) (Bryan et al. 2010); internal commitments involve psychological costs (Thaler 1985, Benabou and Tirole 2004, Bernheim and Rangel 2004, Charness and Gneezy 2009).
Figure 1: The decision timeline with moral hazard.

- **Period 1:**
  - Purchase an insurance contract and infer her effort level at time 1.
  - Allocate her remaining wealth optimally over two periods to maximize her long-run preference.

- **Period 2:**
  - The first-period consumption is realized.
  - The insured risk and the second period consumption are realized.

Choose an effort level to maximize her short-run preference after the first-period consumption is realized.

**Utility function at time 0**

At time 0, the individual endowed with an initial wealth $w$ pays a premium $P$ to obtain an insurance contract that specifies an indemnity $I$ in case of a loss and anticipates her effort level $e \in \{e_h, e_l\}$ at time 1. She then allocates her remaining wealth to optimize the intertemporal consumption utility. At time 1, the first period consumption is realized. At time 2, a loss of size $L$ occurs with probability $p(e)$ and the individual spends all her savings and the indemnity from the insurance contract.

Given $(P, I)$ and $e$, the individual chooses $w_1 \in [P, w - L + I]$ to optimize her intertemporal consumption utility from a long-run perspective

$$U^{LR}(w_1; P, I, e) = u(w_1 - P) - c(e) + \delta[(1 - p(e))u(w - w_1) + p(e)u(w - w_1 - L + I)].$$

(1)

Here $\delta \in (0, 1]$ is the discount factor, $u$ is a continuous differentiable von Neumann-Morgenstern utility function ($u' > 0, u'' < 0$), $w_1$ is the expenditure in period 1 and $c(e)$ is the cost of effort. Denote $p_h \equiv p(e_h) < p_l \equiv p(e_l)$ and $c_h \equiv c(e_h) > c_l \equiv c(e_l)$.

For tractability, we assume throughout the paper that the constraint $w_1 \in [P, w - L + I]$ for (1) is never binding and the unique optimal allocation of (1)

$$w^*_1(P, I, e) = \arg \max_{w_1 \in [P, w - L + I]} U^{LR}(w_1; P, I, e)$$

is determined by the first-order condition. The value $U^{LR}(w^*_1; P, I, e)$ then measures the individual’s long-run preference over the contract $(P, I)$.

**Notation 1:** $IC_e$ denotes the indifference curve for the preference defined by $U^{LR}(w^*_1; P, I, e)$.

**Lemma 1.** In the domain $\{(P, I) : 0 \leq P < I \leq L\}$, the slope of the indifference curve is
given by
\[
\frac{dP}{dI} = \frac{\delta p(e)u'(w - w_1^* - L + I)}{u'(w_1^* - P)}.
\]  
(2)

Moreover,

(i) \( \frac{dP}{dI} \geq p(e) \), with equality holding if and only if \( I = L \);

(ii) single-crossing property: \( \frac{dP}{dI} \) increases with \( p(e) \).

All technical proofs throughout the paper are relegated to the Appendix. \( \frac{dP}{dI} \) measures the marginal premium rate that the individual is willing to pay in order to maintain a given utility level. Formula (2) illustrates two determinants of \( \frac{dP}{dI} \): one is the individual’s perception of loss probability \( p(e) \); the other is the intertemporal marginal rate of substitution, i.e. the ratio of the marginal consumption utility in the loss state in the second period over the marginal consumption utility in the first period. Our study identifies two forces of sophistication along its effects on the two determinants. First, sophistication leads the individual to insure less in order to maintain enough incentive for high effort, thereby decreasing the intertemporal marginal rate of substitution, leading to adverse selection (see Section 4). Second, sophistication helps the individual correctly foresee the timing of her effort in light of procrastination, closing the gap between the perceived and the true loss probability \( p(e) \), leading to advantageous selection (see Section 6).

Utility function at time 1

At time 1 the individual refers to a new short-run preference

\[
U^{SR}(w_1; I, e) = -c(e) + \beta \delta[(1 - p(e))u(w - w_1) + p(e)u(w - w_1 - L + I)].
\]  
(3)

The hyperbolic discounting factor \( \beta \in (0, 1) \) captures that the individual gives a higher weight to the immediate cost incurred at the time of the effort over the delayed gain, undermining her ability to implement her long-run plan.

Graphical tool for equilibrium analysis

All equilibriums are characterized by maximizing individuals’ preferences subject to insurance companies’ zero-profit condition and consumers’ incentive compatibility constraint in
Figure 2: Searching for the first-best contract in the absence of moral hazard.

In the contract space throughout this paper, the horizontal axis represents the indemnity \( I \), the vertical axis represents the premium \( P \). The vertical line \( I = L \) corresponds to full coverage. \( IC_e \) denotes the individual's preference indifference curve with the effort level \( e \). The contracts on the line \( OE \) \((P = p(e)I)\) make zero profit for the insurance companies. In this figure, the utility level of \( IC_e \) is increasing from top to bottom. To obtain the contract \( E \) that maximizes the individual's utility and makes zero profit, we can shift \( IC_e \) upward until it first contacts the zero-profit line \( OE \).

Item (i) of Lemma 1 states that \( E \) in Figure 2 is the actuarially fair contract with full insurance, which reproduces the classical results under the static framework that “a risk-averse individual will choose to fully insure at an actuarially fair premium” (Mossin 1968). Combining this with the single-crossing property given by (ii) of Lemma 1, our dynamic framework enables us to identify the effects of moral hazard and time-inconsistency on the equilibrium and welfare.

2.1 Preference of naifs

Since a naive individual is fully unaware of the self-control problem generated by her time-inconsistent preference, her decision-making process goes as follows:

Step 1: At time 0, after purchasing an insurance contract \((P, I)\), the individual chooses \((w_1^*, e^*)\) to maximize her long-run intertemporal consumption utility \( U^{LR}(w_1^*; P, I, e^*) \).
Step 2: At time 1, the first period consumption $c_1^* = w_1^* - P$ is realized and then the individual decides to exert high effort if and only if $U_{SR}(w_1^*; I, e_h) \geq U_{SR}(w_1^*; I, e_l)$.

Step 3: At time 2, the insured risk and the second period consumption realizes.

**Naifs’ preference at time 0**

Let $w_{1h}^*$ and $w_{1l}^*$ be the shorthand for $w_1^*(P, I, e_h)$ and $w_1^*(P, I, e_l)$ respectively. At time 0, the naive individual’s preference for the contract $(P, I)$ is measured by

$$U_{Nai}(P, I) \equiv \max \{U_{LR}(w_{1h}^*; P, I, e_h), U_{LR}(w_{1l}^*; P, I, e_l)\}.$$

We introduce some notations to describe her preference in the $(P, I)$-coordinate system.

**Notation 2:** Given a point $(a, b)$ and a monotonic locus $l$, $(a, b) \preceq l$ means that $(a, b)$ lies on or to the left of locus $l$; $l \prec (a, b)$ means that $(a, b)$ lies strictly to the right of locus $l$.

**Notation 3:** $P = SL(I)$ denotes the switching locus for the naive individual on which she is indifferent between the two effort levels at time 0.

The switching locus $P = SL(I)$ is determined by

$$U_{LR}(w_{1h}^*; P, I, e_h) = U_{LR}(w_{1l}^*; P, I, e_l).$$ (4)

It is proved in Appendix A.2 that the slope of the locus SL is strictly greater than one. With this notation, naifs’ preference for insurance contracts can be rewritten as

$$U_{Nai}(P, I) = \begin{cases} U_{LR}(w_{1h}^*; P, I, e_h), & \text{if } (P, I) \preceq SL, \\ U_{LR}(w_{1l}^*; P, I, e_l), & \text{if } SL \prec (P, I). \end{cases}$$ (5)

**Notation 4:** $IC_{Nai}$ denotes the preference indifference curve of the naive individual defined by (5).

According to (5), $IC_{Nai}$ coincides with $IC_{e_h}$ when $(P, I) \preceq SL$ and with $IC_{e_l}$ when $SL \prec (P, I)$, and it exhibits a kink on SL where its slope becomes abruptly steeper.
Naifs’ preference at time 1

When $SL < (P, I)$, a naif anticipates to take low effort and allocates $w_1^* = w_{1l}$ in period 1. Since for all $I \leq L$ and all $w_1 \in [P, w - L + I]$, 

$$U^{SR}(w_1; I, e_h) - U^{SR}(w_1; I, e_l) \leq U^{LR}(w_1; P, I, e_h) - U^{LR}(w_1; P, I, e_l),$$

it follows that 

$$U^{SR}(w_{1l}^*; I, e_h) - U^{SR}(w_{1l}^*; I, e_l) \leq U^{LR}(w_{1l}^*; P, I, e_h) - U^{LR}(w_{1l}^*; P, I, e_l) \leq U^{LR}(w_{1h}^*; P, I, e_h) - U^{LR}(w_{1h}^*; P, I, e_l) < 0,$$

which implies that the naif will take low effort as anticipated when time 1 arrives.

When $(P, I) \leq SL$, the conflict between her long-run and short-run preferences arises.

**Notation 5:** $P = SL^{Naif}(I)$ denotes the actual switching locus for the naif on which she is indifferent between the two effort levels at time 1.

To facilitate further analysis, we assume throughout the paper that 

$$\beta < \beta \equiv \min_{(P,I) \leq SL^{TC}} \frac{c_h - c_l}{V(w_{1h}^*; I, e_h) - V(w_{1h}^*; I, e_l) + c_h - c_l}$$

such that the locus $SL^{Naif}$ lies strictly to the left of SL.\textsuperscript{11} Therefore, the switching locus $P = SL^{Naif}(I)$ is determined by 

$$U^{SR}(w_{1h}^*; I, e_h) = U^{SR}(w_{1h}^*; I, e_l).$$

Appendix A.3 proves $\frac{d}{dt} SL^{Naif}(I) > 0$. For the contract $(P, I) \leq SL$, a naif always anticipates at time 0 to take high effort and allocates $w_1^* = w_{1h}$. However, when time 1 arrives, she will take high effort as anticipated if and only if $(P, I) \leq SL^{Naif}$. When $SL^{Naif} < (P, I) \leq SL$, she will actually take low effort, betraying her original plan. This conflict between her long-run perspective and her short-run behavior creates a “preference trap” illustrated in Figure 3.

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\textsuperscript{11}The switching locus $SL^{Naif}$ does not necessarily lie to the left of SL. Since $U^{SR}(w_{1h}^*; I, e_h) - U^{SR}(w_{1h}^*; I, e_l) = \beta[V(w_{1h}^*; I, e_h) - V(w_{1h}^*; I, e_l) + c_h - c_l] - (c_h - c_l)$, when $\beta \geq \max_{(P,I) \leq SL^{TC}} \frac{c_h - c_l}{V(w_{1h}^*; I, e_h) - V(w_{1h}^*; I, e_l) + c_h - c_l} = \overline{\beta}$, all the contracts in the domain $\{(P,I) : (P,I) \leq SL\}$ satisfy the incentive constraint for high effort under a naive individual’s short-run preference. When $\beta \in [\overline{\beta}, \beta)$, part of the locus $SL^{Naif}$ coincides with SL, and the domain $\{(P,I) : SL^{Naif} < (P,I) \leq SL\}$ is nonempty. Numerical studies show that under a wide range of reasonable parameter settings, the value of $\overline{\beta}$ is always close to one. For example, given the parameter setting $u(x) = \frac{e^{1-x}}{1-e^-}$ with $\gamma = 2, \delta = 1, w = 100, L = 40, c_h = 0.0005, c_l = 0, p_h = 0.15, p_l = 0.3, \kappa = 1 - p_h = 0.85$, for any $\beta \leq 0.95$, the locus $SL^{Naif}$ lies strictly to the left of SL. This suggests that Assumption B is not too critical.
Figure 3: A “preference trap” for naifs.

In this figure, SL represents the switching locus on which the naif is indifferent between the two effort levels at time 0, while $SL^{Nai}$ represents the actual switching locus the naif refers to at time 1. The naif’s ex ante long-run utility at some level $k$ is given by indifference curve $(IC^{Nai})$ ABCE. Her ex post welfare at level $k$ (measured by her long-run utility under her actual choice of efforts) is depicted by indifference curve $(WIC^{Nai})$ AB-DF-CE. This figure illustrates a “preference trap” for naifs: when they hold a contract on AB or CE, a new contract in the shadow area tempts them to switch due to their biased belief, and yet makes them worse off.

In Figure 3, a naif’s ex ante long-run utility at level $k$ is given by indifference curve $(IC^{Nai})$ ABCE. Her ex post welfare at level $k$ (measured by her long-run utility under her actual choice of efforts) is depicted by indifference curve $(WIC^{Nai})$ AB-DF-CE. When the insurance contract $(P, I)$ lies on AB or CE, a naif’s long-run effort choice satisfies the incentive constraint for her short-run preference automatically. Therefore, her belief at time 0 about her effort level at time 1 is unbiased and her ex ante utility coincides with her ex-post welfare. However, when the insurance contract $(P, I)$ lies on BC, the naif’s belief becomes biased since she consumes in period 1 as if she were to take high effort but actually takes low effort. Such bias creates a welfare loss: $WIC^{Nai}$ at level $k$, DF, lies below $IC^{Nai}$, BC, implying that contracts on BC generate welfare strictly less than $k$. For a naif who holds a contract on AB or CE, a new contract in the shadow area thus constitutes a “preference trap”: it tempts the naif to switch based on her biased belief and yet makes her worse off.

Note that in our model, naifs’ biased belief in the loss probability is endogenous: naifs are overoptimistic only when $SL^{Nai} \prec (P, I) \preceq SL$ and are objective when $SL \prec (P, I)$ or $(P, I) \preceq SL^{Nai}$. This differentiates our model from those where biased beliefs are exogenously given, such as in Sandroni and Squintani (2007, 2009) and Spinnewijn (2010).
2.2 Preference of sophisticates

A sophisticated individual is fully aware of the self-control problem and derives her utility at time 0 through backward induction:

Step 1: At time 0, given an insurance contract, the sophisticated individual chooses \((w^*_1, e^*)\) and an optimal commitment device to maximize her long-run utility subject to the short-run incentive constraint.

Step 2: At time 1, the first period consumption is realized and she exerts the effort \(e^*\) just as she anticipates at time 0.

Step 3: At time 2, the insured risk and the second period consumption are realized.

When \((P, I) \preceq SL_nai\) or \(SL_nai \prec (P, I)\), all individuals’ long-run effort choice satisfies their short-run incentive constraint naturally. Thus, we only study how sophisticates might behave differently when \(SL_nai \prec (P, I) \preceq SL\).

When \(SL_nai \prec (P, I) \preceq SL\), in order to overcome the self-control problem and prevent themselves from falling into the “preference trap,” sophisticates might resort to a commitment device at time 0. A growing body of field evidences show that individuals invoke commitment devices to help fulfill their long-run plans (Bryan et al. 2010). Consumers may adopt commitment devices by contracting with firms directly. For instance, a consumer can choose underinsurance or insurance contracts with desired incentive clauses such as a rebate check to commit to high precautionary effort. Consumers may also employ personal/private commitment devices. In fact, many anecdotal and field evidences on private commitment devices are available in the area of managing and overcoming addictions (Bernheim and Rangel 2004), which is closely related to life and health insurance markets. For instances, an individual can prepay for a series of personal fitness sessions in order to improve self motivation for exercises; a smoker might purchase self-help books, anti-smoking patches, and pick alternative restaurants to decrease the likelihood of smoking; an individual can also join self-help groups such as Alcoholics Anonymous, Narcotics Anonymous, Gamblers Anonymous, Debtors Anonymous and the like to resist temptation, partially because these social relations raise the cost (social shame) of failure (Battaglini et al. 2005).

In light of Lemma 1, we exploit the two critical roles of commitment devices in our decision framework. One is that they alter the contingent payoff structure of the insurance contract and the intertemporal marginal rate of substitution; the other is that they help a sophisticated individual mitigate procrastination and form an objective perception of loss probability. To capture the two functions of commitment devices in greatly reduced forms,
we focus on two prototypes of commitment devices, respectively. One is incentive lottery—a lottery whose payoff is correlated to the insurable risk. The desire to win the lottery stimulates the individual to take high precautionary effort. The individual has to pay \( \kappa y \) at time 0 to purchase a lottery and gets a reward \( y \) at time 2 if and only if no loss occurs. It is assumed \( \kappa \geq 1 - p_h \) such that the lottery delivers no positive expected return. The other is membership, obtained at a prepaid cost, to help individuals mitigate procrastination. For now we focus on studying the incentive lottery device within our baseline model, and explore how it helps the sophisticated internalize an incentive for high effort. The membership device is studied in Section 5 when we introduce procrastination into our baseline model.

Two broad interpretations can be abstracted from the specific forms of incentive lottery found in practice. In the first case, an incentive lottery is obtained by directly contracting with the insurance company. It can be a reduced amount of insurance coverage or an explicitly included “rebate check” type of contract provision. Alternatively in the second case, the incentive lottery is a commitment device of versatile forms privately adopted by consumers. Both of these cases are of significant practical relevance and have been documented in the literature.\(^{12}\) Although incentive lottery effectively offsets insurance payoff-wise, a key assumption in our model is that we allow consumers to employ private commitment devices in addition to contracting with insurance companies directly. Restricting consumers’ commitment devices to insurance contracts alone will not change the properties of any equilibrium results appearing in Sections 4 and 6 but it does limit the welfare implications. In fact, the key intuition for incentive insurance clauses to improve sophisticates’ welfare in advantageous selection (see Section 6) is that these incentive clauses elicit sophisticates’ private voluntary adoption of incentive lottery and hence can screen them from naifs without making them worse off. For this reason, we focus on the second (broad) interpretation of the incentive lottery throughout the rest of our analysis.

With an incentive lottery, an individual’s long-run and short-run expected utility functions previously given by (1) and (3) are changed to

\[
U^{LR}(w_1, y; P, I, e) = u(w_1 - \kappa y - P) - c(e) \\
+ \delta[(1 - p(e))u(w + y - w_1) + p(e)u(w - w_1 - L + I)],
\]

\[
U^{SR}(w_1, y; I, e) = -c(e) + \beta \delta[(1 - p(e))u(w + y - w_1) + p(e)u(w - w_1 - L + I)]
\]

respectively. By definition, \( U^{LR}(w_1, 0; P, I, e) = U^{LR}(w_1; P, I, e), U^{SR}(w_1, 0; I, e) = U^{SR}(w_1; I, e) \).

**Lemma 2.** When \( \kappa \geq 1 - p_h \), the utility function \( \max_{w_1} U^{LR}(w_1, y; P, I, e) \) is decreasing in \( y \).

---

\(^{12}\)An indirect evidence is offered by Giné et al. (2010). In their paper, an incentive lottery is voluntarily employed by smokers to stop smoking. Smokers are offered an opportunity to open a savings account to giving themselves an incentive to quit. Six months after opening the account, smokers were required to take a urine test, putting their balance on the line if the test showed they had not been able to quit.
Lemma 2 shows that the use of an incentive lottery decreases an individuals’ long-run utility and hence naifs never employ it. However, it provides a positive incentive for sophisticateds to take high effort. Given an insurance contract \((P, I)\) such that \(SL^{\text{Nai}} \prec (P, I) \preceq SL\), we postulate that if a sophisticated individual prefers to take high effort at time 0, she will choose the smallest \(y^*\) such that after allocating the remaining wealth

\[
w_{1hy}^* \equiv \arg \max_{w_1} U^{LR}(w_1, y^*; P, I, e_h),
\]

her short-run incentive to take high effort is satisfied. If she prefers to take low effort at time 0, she takes no incentive lottery and just gets the utility \(U^{LR}(w_{1l}^*, P, I, e_l)\), since the allocation \(w_{1l}^*\) satisfies the short-run incentive naturally.\(^{13}\) In summary, the sophisticated individual’s long-run preference for the contract \(SL^{\text{Nai}} \prec (P, I) \preceq SL\) is

\[
\mathcal{W}^{\text{Sop}}(P, I) \equiv \max \{U^{LR}(w_{1hy}^*, y^*; P, I, e_h), U^{LR}(w_{1l}^*; P, I, e_l)\}.
\]

**Notation 6:** \(P = SL^{\text{Sop}}(I)\) denotes the switching locus for the sophisticated.

The switching locus \(P = SL^{\text{Sop}}(I)\) is defined by

\[
U^{LR}(w_{1hy}^*, y^*; P, I, e_h) = U^{LR}(w_{1l}^*; P, I, e_l).
\]

Noticing that \(SL^{\text{Sop}}\) lies between \(SL^{\text{Nai}}\) and \(SL\), we can rewrite sophisticateds’ preference for insurance contracts as

\[
\mathcal{W}^{\text{Sop}}(P, I) = \begin{cases} 
U^{LR}(w_{1h}^*; P, I, e_h), & \text{if } (P, I) \preceq SL^{\text{Nai}}, \\
U^{LR}(w_{1hy}^*, y^*; P, I, e_h), & \text{if } SL^{\text{Nai}} \prec (P, I) \preceq SL^{\text{Sop}}, \\
U^{LR}(w_{1l}^*; P, I, e_l), & \text{if } SL^{\text{Sop}} \prec (P, I) \preceq \end{cases}
\]

(8)

**Notation 7:** \(IC^{\text{Sop}}\) denotes the preference indifference curve for the sophisticated defined by (8).

Different from naifs, sophisticateds’ preference indifference curve coincides with the ex-post welfare indifference curve. Based on (8), \(IC^{\text{Sop}}\) coincides with \(IC_{e_h}\) when \((P, I) \preceq SL^{\text{Nai}}\) and with \(IC_{e_l}\) when \(SL^{\text{Sop}} \prec (P, I)\). In the domain \(\{(P, I) : SL^{\text{Nai}} \prec (P, I) \preceq SL^{\text{Sop}}\}\), \(IC^{\text{Sop}}\) is in fact a straight line segment whose slope is \(1 - \kappa\), as illustrated in Figure 4. The verification is as follows. From (6) and (7), it is easily seen that choosing \((w_1, P, I)\) with an incentive lottery

\(^{13}\)A rigorous proof is as follows: since \(SL^{\text{Nai}} \prec (P, I) \preceq SL\), we have \(U^{SR}(w_{1l}; I, e_h) - U^{SR}(w_{1l}; I, e_l) < U^{SR}(w_{1h}; I, e_h) - U^{SR}(w_{1h}; I, e_l) < 0\), where the first “\(<\)” follows from \(w_{1l}^* < w_{1h}^*\).
This figure contrasts sophisticates’ preference (IC\textsuperscript{Sop}) with naifs’ (IC\textsuperscript{Nai}). In this figure, SL represents the switching locus on which the naif is indifferent between the two effort levels at time 0; SL\textsuperscript{Nai} represents the actual switching locus the naif refers to at time 1; SL\textsuperscript{Sop} represents the switching locus for the sophisticated. The naif’s preference indifference curve at some level \( k \) is denoted by (IC\textsuperscript{Nai}) \( ABCE \), while the sophisticate’s preference indifference curve at the same level is denoted by (IC\textsuperscript{Sop}) \( AB-D-CE \).

\( y \) is effectively equivalent to choosing \( (w_1 - y, P - (1 - \kappa)y, I - y) \) without such a clause, i.e.,

\[
\begin{align*}
U^{LR}(w_1, y; P, I, e_h) &= U^{LR}(w_1 - y; P - (1 - \kappa)y, I - y, e_h), \\
U^{SR}(w_1, y; I, e_h) &= U^{SR}(w_1 - y; I - y, e_h).
\end{align*}
\]

For a contract \((P, I)\) lying on \(BD\), the incentive lottery \( y^* = \frac{P - P_B}{1 - \kappa} = I - I_B \) together with the allocation \( w_1^* = w_{ih}^*(P_B, I_B) + y^* \) satisfies the minimal short-run incentive and yields the same utility as \((P_B, I_B)\). Since in the domain \( \{(P, I) : SL^{Nai} \prec (P, I) \preceq SL^{Sop}\} \),

slope of IC\textsuperscript{Sop} (=1 - \kappa) \leq p_h < \text{Slope of IC\textsuperscript{Nai}},

sophistication increases the incentive to exert high effort and decreases the marginal premium rate that individuals are willing to pay for increasing the insurance coverage. This force then leads to adverse selection as we illustrate in Section 4.

Assume in the remaining of the paper that \( \kappa = 1 - p_h \), i.e., the incentive lottery is actuarially fairly priced. Hence the segment of IC\textsuperscript{Sop} between SL\textsuperscript{Nai} and SL\textsuperscript{Sop} is parallel to \( P = p_h I \).
3 First-best contracts

We study a competitive insurance market with individuals who differ only in the degree of naivete. As a benchmark for future analysis, this section characterizes the first-best contracts when insurance companies have perfect information regarding an individual’s degree of naivete.

Free entry and perfect competition will ensure that in the first-best equilibrium, the individual achieves her maximal utility subject to the constraint that the insurance company makes zero profit. Formally, the first-best contract for the X-type individual (X=Nai or Sop) is the solution to

$$\max_{P,I} U^X(P,I)$$

subject to

$$P = \begin{cases} phI, & \text{if } (P,I) \preceq SL^X, \\ plI, & \text{if } SL^X \prec (P,I), \end{cases}$$

where $U^{Nai}(P,I)$ and $U^{Sop}(P,I)$ are given by (5) and (8) respectively. In the graphical representation, the first-best contracts are obtained by shifting the preference indifference curve $IC^{Nai}$ or $IC^{Sop}$ upward until it firstly contacts insurance companies’ zero-profit line.

When an individual is naive, there are three types of first-best contracts, illustrated in Figure 5. In all three cases, the naif expects her first-best contract to be $A'$, the intersection of $P = phI$ and SL, at which she plans to take high effort. However, insurance companies cannot offer her the contracts on $AA'$ since they know that she will in fact take low effort on this line. Depending on the size of the gap between $SL^{Nai}$ and SL, the realized first-best contract will be $A$, $E$, or $D$, illustrated in Panels A, B and C of Figure 5 respectively. Note that the size of the gap is positively related to the individual’s present bias (measured by the hyperbolic discounting factor $\beta$).

If the naif’s belief is slightly biased such that the gap between $SL^{Nai}$ and SL is small, her first-best choice is contingent on the cost of high effort. When the cost is low, she prefers to choose the contract $A$, purchasing partial insurance at low premium rate and taking high effort, illustrated in Panel A of Figure 5. When the cost is high, she chooses the contract $E$, purchasing full insurance at high premium rate and taking low effort, illustrated in Panel B of Figure 5. In the above two cases, the naif’s biased belief does not cause a loss to her ex-post welfare.

However, if the naif’s belief is heavily biased such that a large gap between $SL^{Nai}$ and SL arises, she will be tempted into a “preference trap” (see Figure 3), creating a loss to her ex-post welfare. In this case, the naif chooses the contract $D$, purchasing partial insurance at
high premium rate and taking low effort, as shown in Panel C of Figure 5.

When an individual is sophisticated, there are two types of first-best contracts, illustrated in Figure 6. In Panel A, when taking high effort is not too costly, all the contracts on \(AC\)—the segment of \(P = p_h I\) lying in the domain \(\{(P, I) : SL^{\text{Nai}} \prec (P, I) \preceq SL^{\text{Sop}}\}\)—are the first-best choices for the sophisticated. Given a contract on \(AC\), the sophisticated will adopt a corresponding incentive lottery such that when combined with the contract, it yields the same payoff as contract \(A\). In Panel B, when the cost of taking high effort is large, the first-best choice for the sophisticated is \(E\), the contract with full insurance.

4 Second-best equilibrium

This section studies the market equilibrium when the insurance companies cannot distinguish between sophisticates and naïfs. We will formalize the intuition that sophisticates choose to underinsure to provide enough incentive for high effort, leading to a type of adverse selection where the externality caused by unobservable naivete diminishes. We also illustrate another equilibrium where there is a lack of risk-coverage correlation.

Insurance companies are assumed to compete in the RS sense. Formally, a pair of contracts \((P^{\text{Nai}}, I^{\text{Nai}})\) and \((P^{\text{Sop}}, I^{\text{Sop}})\) constitute a RS equilibrium, if the following three conditions are fulfilled:

(1) these contracts are incentive compatible, i.e.,

\[ U^{\text{Nai}}(P^{\text{Nai}}, I^{\text{Nai}}) \geq U^{\text{Nai}}(P^{\text{Sop}}, I^{\text{Sop}}), \quad U^{\text{Sop}}(P^{\text{Sop}}, I^{\text{Sop}}) \geq U^{\text{Sop}}(P^{\text{Nai}}, I^{\text{Nai}}), \]

(2) no contract in the equilibrium makes negative expected profits,

(3) no new contract can be offered and make positive profits.

This definition admits the possibility \((P^{\text{Nai}}, I^{\text{Nai}}) = (P^{\text{Sop}}, I^{\text{Sop}})\), which we term the “pooling equilibrium.”

Next, we show that the first-best contracts illustrated in Figure 5 and Figure 6 satisfy the incentive compatibility naturally and hence they also constitute the RS equilibrium. The key element in the argument is that IC\(^{\text{Nai}}\) coincides with IC\(^{\text{Sop}}\) at the same utility level when \((P, I) \preceq SL^{\text{Nai}}\) and \(SL \prec (P, I)\). The RS equilibriums are depicted in Figure 7.
In this figure, SL represents the switching locus on which the naif is indifferent between the two effort levels at time 0, and SL\textsuperscript{Nai} represents the actual switching locus the naif would take at time 1. IC\textsuperscript{Nai} denotes the naif’s preference indifference curve. Based on the naif’s true effort level, insurance companies offer the zero-profit contracts on the line OA (P = p\textsubscript{h}I) and BE (P = p\textsubscript{l}I). A’ is the intersection of P = p\textsubscript{h}I and SL. This figure illustrates that, the naif expects her first-best contract to be A’, at which she plans to take high effort. However, insurance companies cannot offer her the contracts on AA’ since they know that she will in fact take low effort on this line. Depending on both the size of the gap between SL\textsuperscript{Nai} and SL and the cost of precautionary effort, the realized first-best contract will be A in Panel A, E in Panel B, or D in Panel C respectively.
In this figure, SL\textsuperscript{Sop} represents the switching locus for the sophisticate, and SL\textsuperscript{Nai} represents the actual switching locus for the naif. IC\textsuperscript{Sop} denotes the sophisticate’s preference indifference curve. Based on the sophisticate’s effort level, insurance companies offer the zero-profit contracts on the line OC (\( P = p_hI \)) and BE (\( P = p_lI \)). This figure illustrates that, depending on the cost of taking high effort, the first-best contract can be any contract on AC in Panel A or E in Panel B.

**Case 1:** naifs’ first-best choice is A (Panel A of Figure 5).

In this case, sophisticates’ first-best choice must be A and hence A is a pooling equilibrium, see Panel A of Figure 7.

**Case 2:** naifs’ first-best choice is E (Panel B of Figure 5).

In this case, sophisticates’ first-best choice must be E and hence E is also a pooling equilibrium, which is depicted in Panel B of Figure 7.

In the above two cases, the naifs only have a small bias in their belief such that they make time-consistent decisions on insurance purchase and effort level, resulting in the pooling equilibriums where both naifs and sophisticates behave consistently.

When naifs’ belief is heavily biased such that naifs’ first-best choice is D, sophisticates’ first-best choice can be either AC or E. We only consider the situation where the curve IC\textsuperscript{Nai} passing through D intersects with AC and denote the intersection point by F as illustrated in Panel C and D of Figure 7. Two different market equilibriums follow.

**Case 3:** naifs’ first-best choice is D and sophisticates’ first-best choice is AC (Panel C of Figure 5 and Panel A of Figure 6).

In this case, D and AF satisfy incentive compatibility and constitute the RS equilibrium
In this figure, SL represents the switching locus on which the naif is indifferent between the two effort levels at time 0, $\text{SL}^\text{Nai}$ represents the actual switching locus the naif would take at time 1, and $\text{SL}^\text{Sop}$ represents the switching locus for the sophisticate. $\text{IC}^\text{Nai}$ denotes the naif’s preference indifference curve, and $\text{IC}^\text{Sop}$ denotes the sophisticate’s preference indifference curve. On the line $\text{OA}$, $P = p_h I$, and on $\text{BE}$, $P = p_l I$. This figure illustrates that, when we depict the naif’s first-best choice and the sophisticate’s first-best choice in the same contract space (i.e., when we merge Figure 5 and Figure 6), it turns out that the naif’s first-best choice is incentive compatible with the sophisticate’s first-best choice.
for naifs and sophisticates respectively, as in Panel C of Figure 7. In this equilibrium, adverse selection occurs in the sense that sophisticates take high effort (and become low-risk) and purchase partial insurance with low coverage, whereas naifs take low effort (and become high-risk) and purchase partial insurance with high coverage. Moreover, both sophisticates and naifs achieve their first-best choices, offsetting the externality caused by high-risk individuals and making market intervention redundant. This finding based on our time-inconsistent framework sheds new light on the classic adverse selection model of Rothschild and Stiglitz (1976) where the (unobservable) high-risk individuals lead to negative externality. This distinction arises because in RS’s model with exogenous loss probabilities, the first-best choice for low-risk individuals is full coverage. However, in our model, the first-best choice for low-risk individuals is partial insurance for them to maintain enough incentive to overcome the self-control problem and make high effort.

Case 4: naifs’ first-best choice is D and sophisticates’ first-best choice is E (Panel C of Figure 5 and Panel B of Figure 6).

In this case, D and E are incentive compatible and become the RS equilibrium, as illustrated in Panel D of Figure 7. In this equilibrium, both sophisticates and naifs are high-risk, and yet naifs obtain less insurance coverage due to their biased belief that they will exert high effort. Therefore, this equilibrium predicts no correlation between the risk type and the insurance coverage, as observed in some insurance markets (see Chiappori and Salanié 2000 and Saito 2006 for the evidence in auto insurance markets).

In summary, when the unobservable heterogeneity lies in the degree of naivete, sophisticates leverage a commitment device to insure less and maintain an incentive to take high effort and subsequently become low-risk. This motivation allows sophisticates to achieve their first-best choice even after satisfying the self-selection constraint of naifs. Although in practice the specific commitment device may take on forms different from the one described in this paper, the intuition remains the same to a large extent. The above equilibrium results are summarized in Proposition 1.

Proposition 1. Assume that the unobservable heterogeneity in consumers lies in the degree of naivete and sophisticates employ some incentive lottery device to overcome the self-control problem. Then the first-best insurance contracts for sophisticates and naifs are incentive compatible, constituting the RS equilibriums, which can exhibit pooling, adverse selection, or a lack of risk-coverage correlation.

\footnote{For example, \( \kappa \) can be different from \( 1 - p_h \); the incentive lottery can be non-perfectly correlated to the insured risk; and the individual can adjust her intertemporal consumption to create more incentives for the short-run self, etc.}
5 Effects of procrastination

In the previous section, we have analyzed one force of sophistication when individuals face only the choice of low or high effort. We show that sophistication can provide incentive for high effort and decrease the marginal premium rate the individuals are willing to pay. This section will identify another force that sophistication prevents individuals from optimistic underestimation of loss probability when they are given the opportunity to procrastinate making high effort. This force increases the marginal premium rate the individuals are willing to pay, leading to the formation of advantageous selection.

Now we introduce to our baseline model flexibility for individuals to choose not only high or low effort, but also when to take the high effort. In this case, individuals with time-inconsistent preference tend to procrastinate taking high effort. Sophisticates can foresee their procrastination (and associated negative consequences) and hence will employ some commitment devices to mitigate it, while naifs cannot. To formalize this intuition, we make a key assumption that the sooner a precautionary activity is completed with high effort, the more efficient it is in reducing the loss probability. In many insurance cases, the efficiency of the precautionary activity depends crucially on when it is initiated. For example, performing timely maintenance of a car can substantially lower the cost and the probability of a car accident. Similarly, the success of preventive care measures hinges largely on timely implementation and that is an important contributing factor to the costs and outcomes of future treatments.

To analyze time-inconsistent individuals’ choice of timing in taking high effort, we further divide period 2 into subperiods. The payoff of the precautionary activity along with the timeline is illustrated in Figure 8. There are $N + 1$ ($N \geq 1$) time points in period 2 at which the individual can choose to complete the activity with high effort. If the individual chooses to complete the activity at time $1-K$, the total cost will be $c_h^{(K)}$ and the loss probability is reduced to $p_h^{(K)}$. To capture that the efficiency of the precautionary activity decreases as it is being postponed, we assume

\[ c_l < c_h \equiv c_h^{(0)} < c_h^{(1)} < \cdots < c_h^{(N)}, \quad p_h \equiv p_h^{(0)} < p_h^{(1)} < \cdots < p_h^{(N)} < p_l. \]

Note that low effort can be taken at any time with the same cost $c_l$ and the associated loss probability is $p_l$.

We now introduce a new “membership” commitment device to help the sophisticates mitigate procrastination. This is because the original incentive lottery device only changes the final payoff at the end of the second period, and thus is only relevant to taking high effort or not, rather than when to take the effort. With a reasonable parameter setting, Proposition
A in Appendix A.5 illustrates that individuals procrastinate taking high effort however the incentive lottery is designed, justifying the use of a new form of commitment device. We assume this new device takes the form of a membership. At time 0, individuals can obtain the membership at a negligible cost to motivate high effort. If the individual cannot keep her promise to take high effort before the time point $1-K$, she will incur a significant monetary or mental cost $\varepsilon_K$. This intuitively plausible assumption characterizes that the monetary or the mental cost arising from procrastination is usually low at the initial stage but will eventually become very high. This type of membership device is commonly observed in practice, e.g., self-punishment (Bryan et al. 2010), alcohol clinics, Christmas clubs, fat farms (Laibson 1997), etc. Since the membership is costly if the activity is not finished on time, only sophisticates can foresee the value of the membership and employ it to mitigate procrastination.

Under the assumption above, Proposition A in Appendix A.5 characterizes that

(A1) Naifs hold the wrong belief that they will complete the precautionary activity with high effort at time point 1-0 and the loss probability will be $p_h$, but in fact they always procrastinate until they finally give up making high effort.

(A2) Sophisticates will procrastinate at the initial subperiod since the cost of procrastination is not significant at this stage. However, they can foresee their tendency to procrastinate and will invoke a membership to stop procrastination at some time point $1-m$ ($m \in \{1, \cdots, N\}$) when the cost of procrastination becomes sufficiently large. In the meanwhile, sophisticates hold the correct belief that they will complete the precautionary activity with high effort only at an intermediary time point $1-m$ in the second period, and the associated loss probability will be $p_{h}^{(m)}$.

Sophisticates realize that they can only mitigate, not completely overcome, procrastination. Therefore, sophistication helps them form an unbiased estimation of loss probability $p_{h}^{(m)}$, which is larger than $p_h$. Relative to naifs, this increases the marginal premium rate.
sophisticates are willing to pay for the increased insurance coverage, leading to the second-best equilibrium in which sophisticates take high effort (and become low-risk) and purchase partial insurance with high coverage, whereas naifs take low effort (and become high-risk) and purchase partial insurance with low coverage, i.e., advantageous selection.

Two remarks are in order. First, the two prototypes of commitment devices studied in our paper— incentive lottery and membership—play distinct roles in overcoming self-control problem. Incentive lottery helps sophisticates maintain the incentive to take high effort, without any effect on when to take the effort, while membership helps them make the effort at the promised time point. In the following section, we assume that sophisticates have access to both devices: they will use the membership to mitigate procrastination as long as they decide to take high effort, i.e., as long as \((P, I) \preceq \text{SL}^{\text{Sop}}\), and will go on to invoke an incentive lottery additionally to maintain the incentive to take high effort when the utility of doing so is higher than that of switching to low effort. The key purpose for such an assumption is to retain the shape of \(IC^{\text{Sop}}\) as in Figure 4 and produce graphical illustrations consistent with previous analysis.\(^{15}\)

Second, as we have explained previously, the incentive lottery or membership is merely a simplification of a versatile collection of commitment devices observed in reality. Our equilibrium results are not driven by these specific forms of commitment devices, but rather by the two forces of sophistication manifested in the commitment devices.

6 Advantageous selection

Consider the situation described by (A1) and (A2) in the previous section. Sophisticates take more effort (and thus become low-risk) and at the same time purchase more insurance than naifs since they are less optimistic toward the precautionary activity. Advantageous selection then arises.

**Proposition 2.** Suppose that naifs are partially insured at high premium rate in the first-best contract, i.e., contract D in Panel C of Figure 5. Let \(F\) be the intersection point of the curve \(IC^{\text{Nai}}\) and \(\text{SL}^{\text{Sop}}\) (see Figure 9). Assume that

\[
\begin{align*}
(\text{i}) & \quad \text{the curve } IC^{\text{Sop}} \text{ passing through } F \text{ lies below } OE; \\
(\text{ii}) & \quad p_h^{(m)} \text{ is large enough such that at point } F, IC^{\text{Sop}} \text{ is steeper than } IC^{\text{Nai}}
\end{align*}
\]

\(^{15}\)An alternative assumption that sophisticates only adopt the membership device does not alter the results developed in this section.
In this figure, SL represents the switching locus on which naifs are indifferent between the two effort levels at time 0, and SL\textsubscript{Sop} represents the switching locus for sophisticates. IC\textsuperscript{Nai} denotes naifs’ preference indifference curve, and IC\textsuperscript{Sop} denotes sophisticates’ preference indifference curve. \(P = p_l I\) on the line \(OE\), \(P = p_h^{(m)} I\) on \(OB\), and on the “fair pooling line,” \(P = [(1 - \lambda) p_h^{(m)} + \lambda p_l] I\). This figure illustrates that naifs obtain their first-best contract \(D\) while sophisticates obtain their second-best contract \(F\), forming the advantageous selection equilibrium.
(iii) the fraction of naifs $\lambda$ in the population is substantially large so that the fair pooling line described by

$$P = [(1 - \lambda)p_h^{(m)} + \lambda p_l]I$$

lies above the curve $IC^{Sop}$ passing through $F$.

Then there exists a unique RS equilibrium, in which naifs purchase insurance at $D$ and take low effort, while sophisticates purchase insurance at $F$ and take high effort.

**Proof:** Notice that no pooling equilibrium on the fair pooling line exists, since the contract $F$ is always more attractive to sophisticates than any insurance contract lying on the fair pooling line. Next, we show that the pair of contracts $D$ and $F$ form the unique RS equilibrium. Offering contracts below the curve $IC^{Sop}$ passing through $F$ is loss making, since these contracts attract both sophisticates and naifs, and yet lie below the fair pooling line. Offering contracts lying between the curve $IC^{Nai}$ passing through $D$ and the curve $IC^{Sop}$ passing through $F$ in the domain $\{(P, I) : (P, I) \preceq SL^{Sop}\}$ is also loss making since only naifs are attracted. Observe that $D$ is the first-best contract for naifs and there exist no other separating contracts which could not only satisfy incentive compatibility but also improve sophisticates’ welfare. Q.E.D.

The condition (ii) of Proposition 2 implies that sophisticates cannot take high effort until a substantially deferred time point and this condition is satisfied if the cost of procrastination increases slowly. When the cost of procrastination increases rapidly, two other types of equilibriums can arise. When the cost of taking high effort is not too large such that the utility of doing so exceeds the utility of giving up, sophisticates will take high effort at an early time. In this case, sophisticates will recognize that their loss probability is low. Combining this with their motivation to maintain enough incentive for exerting high effort, sophisticates will prefer to purchase less insurance than naifs who take low effort and are offered the contact $D$ in Figure 9. This leads to adverse selection. When the cost of taking high effort is large enough such that the switching locus $SL^{Sop}$ in Figure 9 is pushed sufficiently to the left, sophisticates will switch to low effort and purchase full insurance at high premium rate, leading to the second-best equilibrium which predicts no correlation between the risk type and the insurance coverage.

The condition (iii) of Proposition 2 indicates that in order to observe advantageous selection, the fraction of naifs in the population should be large. When the fraction of naifs is small, a pooling RS equilibrium may appear on the fair pooling line in the same spirit of Position 2 in Sonnenholzner and Wambach (2009). Since this scenario does not provide new insights into the welfare analysis presented below, we do not study it here. Note that Sonnenholzner and Wambach (2009) obtain advantageous selection based on the intuition that a
more patient individual will make more precautionary effort (and thus becomes low-risk) and purchase more insurance. In their model, patience is exogenously given and all individuals have the same estimation of loss probability. In contrast, \( p^{(m)}_h > p_h \) is a necessary condition for advantageous selection in our model, and only sophisticates can mitigate procrastination and correctly estimate the true loss probability.

When the market exhibits advantageous selection, market solutions are available to alleviate the externality caused by unobservable naivete.

Firstly, we find that traditional public polices promoting cross-subsidization from low-risk to high-risk individuals such as compulsory insurance and taxes are welfare enhancing.\(^\text{16}\) In Figure 10, we note the original equilibrium \( D \) and \( F \) and the ex-post welfare indifference curve for naifs \( WIC^{\text{Nai}} \) passing through \( D \). Suppose compulsory insurance is offered at \( D' \) on the fair pooling line, where \( D' \) lies to the right of \( WIC^{\text{Nai}} \) passing through \( D \). Beyond the compulsory insurance \( D' \), consumers can purchase additional coverage in a competitive private insurance market. The zero-profit lines for the private insurance companies correspond to the dash lines which originate from \( D' \) and parallel to \( OB \) and \( OE \). In the same spirit of Proposition 2, in the private insurance market, naifs never purchase any additional coverage, while sophisticates will increase their coverage to \( F' \). Since \( D' \) improves naifs’ welfare relative to \( D \) and \( F' \) improves sophisticates’ welfare relative to \( F \), the new equilibrium under the intervention policy is a Pareto improvement from the original equilibrium \( D \) and \( F \).

Secondly, we show that certain incentive clauses in insurance contracts can serve as a welfare enhancing screening device. In Figure 11, when insurance contracts contain no incentive clauses, the RS equilibrium is achieved at the pair of contracts \( D \) and \( F \): naifs purchase \( D \) and take low effort, whereas sophisticates purchase \( F \) and take high effort by employing an incentive lottery and a membership. In this equilibrium, the employment of these commitment devices by sophisticates is a self-invoked hidden action. If an incentive clause such as a rebate check or a membership (or both) is added to insurance contracts, the impact will be twofold: on the one hand, since naifs never acknowledge the value of such devices, their new indifference curve \( IC'^{\text{Nai}} \) at the same utility level as originally given by \( D \) (without any additional clauses) lies below the curve \( IC^{\text{Nai}} \) passing through \( D \); on the other hand, since sophisticates can exactly offset the effect of the incentive clause by eliminating their own effort in invoking a similar commitment device privately, their new indifference curve at the same utility level as that originally derived from \( F \) (without any additional clauses) coincides exactly with the curve \( IC^{\text{Sop}} \) passing through \( F \) in the domain \( \{(P, I) : (P, I) \preceq SL^{\text{Sop}}\} \). Therefore, if insurance companies offer two contracts, one is at \( D \), which only specifies the premium and coverage, and the other lies in the shadow area, which contains the incentive clause, then naifs will keep

\(^{16}\)The policies of compulsory insurance and taxes are in effect the same concerning cross-subsidization, see Wilson (1977), Dahlby (1981) and Crocker and Snow (1985).
Figure 10: Use of compulsory insurance under “advantageous selection.”

Notations in this figure follow those in Figure 9. The original equilibrium is given by $D$ and $F$. The compulsory insurance is offered at $D'$ on the fair pooling line. The zero-profit lines for the private insurance companies correspond to the dash lines which originate from $D'$ and parallel to $OB$ and $OE$. This figure illustrates that, after obtaining the compulsory insurance contract $D'$, naifs never purchase any additional coverage in the private insurance market while sophisticates choose to increase their coverage to $F'$. The new equilibrium $D'$ and $F'$ is a Pareto improvement from the original equilibrium $D$ and $F$. 
Figure 11: Use of self-selected incentive insurance clauses under “advantageous selection.”

Notations in this figure follow those in Figure 9. The original equilibrium is given by $D$ and $F$. When an incentive clause is added to insurance contracts, naifs’ new indifference curve $IC'_{Nai}$ at the same utility level as $IC_{Nai}$ lies below $IC_{Nai}$, while sophisticates’ new indifference curve at the same utility level as $IC_{Sop}$ does not change. This figure illustrates that, if insurance companies offer two contracts, one is at $D$, which only specifies the premium and coverage, and the other lies in the shadow area, which contains the incentive clause, then naifs will keep their original choice $D$, but sophisticates will switch to the one in the shadow area, which is Pareto superior to $F$.

It is worth noting that if the self-selected incentive clause becomes compulsory, it can no longer increase all individuals’ welfare, as illustrated in Figure 12. In fact, in the $(P,I)$-coordinate system, introducing an actuarially fair compulsory rebate check with $\kappa = (1 - \lambda)p_h + \lambda p_l$ is equivalent to offsetting the corresponding amount of insurance, i.e., moving the coordinate origin $O$ along the fair pooling line $P = [(1 - \lambda)p_h^{(m)} + \lambda p_l]I$ to a new coordinate origin $O'$. It is easily seen that compulsory rebate check has an effect opposite to that of compulsory insurance, i.e., it makes all individuals worse off.

7 Concluding remarks

This paper examines the effects of the externality caused by unobservable consumer naivete on market equilibrium and social welfare.

We identify two distinct forces of sophistication driving the formation of market equilibrium. An individual with time-inconsistent preference encounters self-control problem that undermines her precautionary activity in two ways: she is tempted to take low precautionary
Figure 12: Use of compulsory rebate check under “advantageous selection.”

Notations in this figure follow those in Figure 9. The original equilibrium is given by $D$ and $F$. Introducing an actuarially fair compulsory rebate check with $\kappa = (1 - \lambda)p_h + \lambda p_l$ is equivalent to offsetting the corresponding amount of insurance, i.e., moving the coordinate origin $O$ along the fair pooling line $P = [(1 - \lambda)p_h^{(m)} + \lambda p_l]I$ to a new coordinate origin $O'$. This figure illustrates that compulsory rebate check has an effect opposite to that of compulsory insurance and it makes all individuals worse off.

effort when she has a choice of taking high effort or not; or she is induced to procrastinate taking high effort when she has the choice of when to take high effort. The two forces of sophistication play a prominent role in overcoming each of the above self-control problem and they are manifested by the sophisticates’ voluntary adoption of appropriate commitment devices (through direct contracting with the insurance company or privately selected devices). One force is that sophistication helps individuals internalize the incentive to take high effort by use of commitment devices, and hence sophisticates are willing to pay a lower premium to increase insurance coverage than naifs, predicting adverse selection. The other is that sophistication helps prevent the optimistic underestimation of loss probability, and in this case sophisticates are willing to pay a higher premium to increase insurance coverage than naifs, resulting in advantageous selection. Moreover, we show that there exists an equilibrium in which sophisticates take the same effort (and hence have the same risk) as naifs but purchase more insurance than naifs, resulting in the lack of risk-coverage correlation. We also find pooling equilibriums in which sophisticates and naifs take the same effort and purchase the same amount of insurance coverage. Therefore our time-inconsistent framework provides a comprehensive explanation for different risk-coverage correlations observed in various insurance markets.

Two new welfare implications are that when adverse selection appears, sophisticates choose to adopt some commitment devices to maintain enough incentive, which eliminates externality rendering market intervention such as compulsory insurance irrelevant; when advantageous selection appears, some incentive insurance clauses such as self-selected rebate clauses.
check can be welfare-enhancing.

Finally, our analysis indicates two potential paths for future research. First, developing a similar analysis in a monopolistic insurance market to better understand hidden consumer naivete. Second, our results point to new avenues for empirically testing whether unobservable heterogeneity in the degree of naivete drives market selection. Our models and results developed in the context of the insurance market can also broadly apply to other financial markets or the labor market.

Appendix

A.1. Proof of Lemma 1. The first-order optimality condition of the program (2) writes

\[ u'(w_1^* - P) - \delta[(1 - p)u'(w - w_1^*) + pu'(w - w_1^* - L + I)] = 0, \]  

(A1)

which implicitly determines a function \( w_1^* = w_1^*(P, I, p) \). For later use, we differentiate (A1) with respect to \( P, I \) and \( p \) respectively to get

\[
\frac{\partial w_1^*}{\partial P} = \frac{u''(w_1^* - P)}{u''(w_1^* - P) + \delta(1 - p)u''(w - w_1^*) + \delta pu''(w - w_1^* - L + I)},
\]

(A2)

\[
\frac{\partial w_1^*}{\partial I} = \frac{\delta pu''(w - w_1^* - L + I)}{u''(w_1^* - P) + \delta(1 - p)u''(w - w_1^*) + \delta pu''(w - w_1^* - L + I)},
\]

(A3)

\[
\frac{\partial w_1^*}{\partial p} = \frac{\delta[u'(w - w_1^* - L + I) - u'(w - w_1^*)]}{u''(w_1^* - P) + \delta(1 - p)u''(w - w_1^*) + \delta pu''(w - w_1^* - L + I)}.
\]

(A4)

To prove (i), differentiating the equation \( U^{LR}(w_1^*(P, I, p); P, I, p) \equiv k \) with respect to \( I \), where \( k \) is an arbitrary positive constant, making use of (A1), and applying the envelope theorem, we obtain

\[-u'(w_1^* - P) \frac{dP}{dI} + \delta pu'(w - w_1^* - L + I) = 0, \]

(A5)

or equivalently,

\[
\frac{dP}{dI} = \frac{\delta pu'(w - w_1^* - L + I)}{u'(w_1^* - P)} = \frac{pu'(w - w_1^* - L + I)}{(1 - p)u'(w_1^* - w_1^*) + pu'(w - w_1^* - L + I)}.
\]

(A6)

where (A6) follows by inserting (A1) into (A5). The last expression in (A6) justifies that \( \frac{dP}{dI} \geq p \), with equality holding if and only if \( I = L \). Next, we differentiate (A5) implicitly with
respect to $I$ to get

$$u'(w_1^*-P)\frac{d^2P}{dI^2} = u''(w_1^*-P)\left(\frac{dP}{dI}\right)^2 + \delta pu''(w-w_1^*-L+I) - \left[u''(w_1^*-P)\frac{dP}{dI} + \delta pu''(w_1-w_1^*-L+I)\right] \left(\frac{\partial w_1^*}{\partial P} \frac{dP}{dI} + \frac{\partial w_1^*}{\partial I}\right).$$

(A7)

Inserting (A2) and (A3) into (A7), we see that $\frac{d^2P}{dI^2}$ has the same sign as

$$S = - \left[u''(w_1^*-P)\left(\frac{dP}{dI}\right)^2 + \delta pu''(w-w_1^*-L+I)\right] \times \left[u''(w_1^*-P) + \delta(1-p)u''(w-w_1^*) + \delta pu''(w-w_1^*-L+I)\right]$$

$$+ \left[u''(w_1^*-P)\frac{dP}{dI} + \delta pu''(w_1-w_1^*-L+I)\right]^2.$$

Since

$$S \leq - \left[u''(w_1^*-P)\left(\frac{dP}{dI}\right)^2 + \delta pu''(w-w_1^*-L+I)\right] \times \left[u''(w_1^*-P) + \delta pu''(w-w_1^*-L+I)\right]$$

$$+ \left[u''(w_1^*-P)\frac{dP}{dI} + \delta pu''(w_1-w_1^*-L+I)\right]^2 = - \delta pu''(w_1^*-P)u''(w_1-w_1^*-L+I)\left(\frac{dP}{dI} - 1\right)^2 \leq 0,$$

it proves that $\frac{d^2P}{dI^2} \leq 0$. To prove (ii), by use of the first expression of (A6), we get

$$[u'(w_1^*-P)]^2 \frac{d}{dp} \left(\frac{dP}{dI}\right) = \delta u'(w-w_1^*-L+I)u'(w_1^*-P) - \delta p[u''(w-w_1^*-L+I)u'(w_1^*-P) + u'(w-w_1^*-L+I)u''(w_1^*-P)] \frac{\partial w_1^*}{\partial p}. \quad (A8)$$
Inserting (A4) into (A8), we see that $\frac{d}{dp} \left( \frac{dP}{dl} \right)$ has the same sign as

$$
C = -u'(w - w^*_1 - L + I)u'(w^*_1 - P) \\
\times \left[ u''(w^*_1 - P) + \delta(1-p)u''(w - w^*_1) + \delta pu''(w - w^*_1 - L + I) \right] \\
+ \delta p[u''(w - w^*_1 - L + I)u'(w^*_1 - P) + u'(w - w^*_1 - L + I)u''(w^*_1 - P)] \\
\times \left[ u'(w - w^*_1 - L + I) - u'(w - w^*_1) \right] \\
= -u''(w^*_1 - P)u'(w - w^*_1 - L + I) \\
\times \left[ u'(w^*_1 - P) - \delta pu'(w - w^*_1 - L + I) + \delta pu'(w - w^*_1) \right] \\
- \delta u'(w^*_1 - P) \\
\times [(1-p)u'(w - w^*_1 - L + I)u''(w - w^*_1) + pu'(w - w^*_1)u''(w - w^*_1 - L + I)].
$$

Substituting $u'(w^*_1 - P)$ from (A1), we are led to

$$
C = -\delta u'(w - w^*_1)u'(w - w^*_1 - L + I)u''(w^*_1 - P) \\
- \delta(1-p)u'(w^*_1 - P)u'(w - w^*_1 - L + I)u''(w - w^*_1) \\
- \delta pu'(w^*_1 - P)u'(w - w^*_1)u''(w - w^*_1 - L + I)
$$

which proves $\frac{d}{dp} \left( \frac{dP}{dl} \right) > 0$. \qed

A.2. **Proof of the fact** $\frac{d}{dl} SL(I) > 1$. Let $w^*_1h$ and $w^*_1t$ be the shorthand for $w^*_1h(P, I)$ and $w^*_1t(P, I)$ respectively. By definition, $w^*_1h$ and $w^*_1t$ satisfy

$$
u'(w^*_1h - P) - \delta [(1 - p_h)u'(w - w^*_1h) + p_hu'(w - w^*_1h - L + I)] = 0, \quad (A9)
$$

$$
u'(w^*_1t - P) - \delta [(1 - p_l)u'(w - w^*_1t) + p_lu'(w - w^*_1t - L + I)] = 0, \quad (A10)
$$

and (4) writes

$$
u(w^*_1h - P) - c_h + \delta [(1 - p_h)u(w - w^*_1h) + p_hu(w - w^*_1h - L + I)] \\
= u(w^*_1h - P) - c_l + \delta [(1 - p_l)u(w - w^*_1h) + p_lu(w - w^*_1h - L + I)], \quad (A11)
$$

which determines the implicit function $P = SL^{TC}(I)$. Differentiating (A11) with respect to $I$, making use of (A9) and (A10), and applying the envelope theorem, we get

$$
[u'(w^*_1t - P) - u'(w^*_1h - P)] \frac{dP}{dl} = \delta [p_lu'(w - w^*_1l - L + I) - p_hu'(w - w^*_1h - L + I)].
$$

Substituting $u'(w^*_1h - P), u'(w^*_1t - P)$ from (A9), (A10), we arrive at $\frac{dP}{dl} = \frac{A}{A+B}$, where

$$
A = p_lu'(w - w^*_1l - L + I) - p_hu'(w - w^*_1h - L + I),
$$

$$
B = (1 - p_l)u'(w - w^*_1l) - (1 - p_h)u'(w - w^*_1h).
$$

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Given \( P, I \), we have for \( w^*_1(P, I, p) \) that

\[
\frac{d}{dp}[p\prime(w - w^*_1 - L + I)] = u'(w - w^*_1 - L + I) - pu''(w - w^*_1 - L + I)\frac{\partial w^*_1}{\partial p}. \tag{A12}
\]

Inserting (A4) into (A12), we see that \( \frac{d}{dp}[p\prime(w - w^*_1 - L + I)] \) has the same sign as

\[
\mathcal{D} = -u'(w - w^*_1 - L + I)[u''(w^*_1 - P) + \delta(1 - p)u''(w - w^*_1) + \delta pu''(w - w^*_1 - L + I)]
+ \delta pu''(w - w^*_1 - L + I)[u'(w - w^*_1 - L + I) - u'(w - w^*_1)]
= -u'(w - w^*_1 - L + I)u''(w^*_1 - P) - \delta(1 - p)u'(w - w^*_1 - L + I)u''(w - w^*_1)
- \delta pu'(w - w^*_1)u''(w - w^*_1 - L + I) > 0.
\]

Since \( p_l > p_h \), it follows that \( A > 0 \). Observe that for any given \( I < L \), we have

\[
\frac{d}{dw_1}U^{\text{LR}}(w_1; P, I, e_h) = u'(w_1 - P) - \delta[(1 - p_h)u'(w - w_1) + p_hu'(w - w_1 - L + I)]
> u'(w - P) - \delta[(1 - p_l)u'(w - w_1) + p_lu'(w - w_1 - L + I)]
= \frac{d}{dw_1}U^{\text{LR}}(w_1; P, I, e_l). \tag{A13}
\]

The single-crossing property (A13) implies \( w^*_{1h} > w^*_{1l} \), which gives \( B < 0 \) and \( A + B = [u'(w^*_{1l} - P) - u'(w^*_{1h} - P)] > 0 \). Therefore \( \frac{dP}{dI} = \frac{A}{A + B} > 1 \). \( \square \)

A.3. Proof of the fact \( \frac{d}{dI}\text{SL}^{\text{Nai}}(I) > 0 \). The switching locus \( \text{SL}^{\text{Nai}} \) is determined by

\[
u(w - w^*_{1h}) - u(w - w^*_{1h} - L + I) = \frac{1}{\beta\delta} \left( \frac{c_h - c_l}{p_l - p_h} \right), \tag{A14}
\]

where \( w^*_{1h} \) satisfies (A9). Differentiating (A14) with respect to \( I \), making use of (A9), and applying the envelope theorem, we get

\[
[u'(w - w^*_{1h} - L + I) - u'(w - w^*_{1h})] \frac{\partial w^*_{1h}}{\partial P} \frac{dP}{dI} = u'(w - w^*_{1h} - L + I) - [u'(w - w^*_{1h} - L + I) - u'(w - w^*_{1h})] \frac{\partial w^*_{1h}}{\partial I}. \tag{A15}
\]

Due to the fact \( \frac{\partial w^*_{1h}}{\partial p} > 0 \) (recalling (A2)) and inserting (A3) into the righthand side of (A15), it is easily seen that \( \frac{dP}{dI} \) has the same sign as \( \mathcal{D} \), with the \( w^*_1 \) therein replaced by \( w^*_{1h} \). Hence there holds \( \frac{dP}{dI} > 0 \). \( \square \)

A.4. Proof of Lemma 2. The proof proceeds by two steps. Firstly, we consider the case
when the effort level is exogenous. The optimal allocation

$$w^*_1(P, I, p(e), y) = \arg \max_{w_1} U_{LR}(w_1, y; P, I, e)$$  \hspace{1cm} (A16)$$

can be solved by the first-order condition

$$u'(w^*_1 - \kappa y - P) - \delta[(1 - p(e))u'(w + y - w^*_1) + p(e)u'(w - w^*_1 - L + I)] = 0.$$ 

Following the envelop theorem, we get

$$\frac{\partial}{\partial y} U_{LR}(w^*_1, y; P, I, e) = -u'(w^*_1 - \kappa y - P)\kappa + \delta[(1 - p(e))(1 - \kappa)u'(w + y - w^*_1) - \kappa p(e)u'(w - w^*_1 - L + I)] < 0,$$

provided $\kappa \geq 1 - p(e)$. Secondly, let $w^*_{1h}$ and $w^*_{1l}$ be the shorthand for $w^*_1(P, I, p(e_h), y)$ and $w^*_1(P, I, p(e_l), y)$ respectively. Since we have proved in the above that both $U_{LR}(w^*_{1h}, y; P, I, e_h)$ and $U_{LR}(w^*_{1l}, y; P, I, e_l)$ are decreasing in $y$, it is obvious that the optimal value of the optimization problem

$$\max \left\{ \max_{w_1} U_{LR}(w_1, y; P, I, e_h), \max_{w_1} U_{LR}(w_1, y; P, I, e_l) \right\}$$

is also decreasing in $y$. \hfill \square

A.5. Proposition A. Suppose $\delta = 1$ and individuals have the options either to complete the precautionary activity with high effort or delay the activity to the next subperiod. Assume a hyperbolic discounting parameter $\beta'$ measures the present-bias in the subperiods 1-0 to 1-$N$ ($\beta' \in (0, 1)$ is not necessarily equal to $\beta$). The membership is designed such that if the individual cannot complete the activity with high effort before the time point 1-$K$, she will incur a cost $\varepsilon_K$ ($K = 0, \ldots, N$). If

$$\frac{c_h^{(N)} - c_l^{(K)}}{\beta'} > \left(p_l - p_h^{(N)}\right) \left[u\left(\frac{w}{2}\right) - u\left(\frac{w}{2} - L\right)\right],$$  \hspace{1cm} (A17)$$

$$\zeta^{(K)} = \frac{c_h^{(K)} - c_l^{(K+1)}}{\beta'} - \left(c_h^{(K+1)} - c_l\right) - \left(p_h^{(K+1)} - p_h^{(K)}\right) \left[u\left(\frac{w}{2}\right) - u\left(\frac{w}{2} - L\right)\right] > 0, \hspace{1cm} (A18)$$

for all $K = 0, \ldots, N$ and

$$\varepsilon_0 = \cdots = \varepsilon_{m-1} = 0, \quad \varepsilon_m > \beta' \left(p_l - p_h^{(m+1)}\right) \left[u\left(\frac{w}{2}\right) - u\left(\frac{w}{2} - L\right)\right],$$  \hspace{1cm} (A19)$$

for an integer $m$ within 1, $\ldots, N - 1$, we have:
(i) naifs never take high effort but believe they would behave themselves in the subperiod 1-0 to 1-N when \((P, I) \leq \text{SL}\);

(ii) sophisticates complete the activity with high effort at the time point 1-m.

**Proof:** From a long-run perspective, the benefit of completing the precautionary activity with high effort at time \(1-K\), \(K = 0, 1, \ldots, N\), is given by

\[
\delta \left[ (1 - p_h^{(K)}) u(w - w_1) + p_h^{(K)} u(w - w_1 - L + I) \right]
\]

the consumption utility at time 2 with high effort

\[
- \delta [(1 - p_l) u(w - w_1) + p_l u(w - w_1 - L + I)]
\]

the consumption utility at time 2 with low effort

\[
= \delta \left( p_l - p_h^{(K)} \right) [u(w - w_1) - u(w - w_1 - L + I)].
\]

Following the terminology in O’Donoghue and Rabin (1999) (cf., pp108-109), the “reward schedule” and “cost schedule” for the individual to complete the activity are given by

\[
\mathcal{V} = (\mathcal{V}_0, \mathcal{V}_1, \ldots, \mathcal{V}_N), \text{ where } \mathcal{V}_K = \delta \left( p_l - p_h^{(K)} \right) [u(w - w_1) - u(w - w_1 - L + I)]
\]

and \(\mathcal{C} = (\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_N)\), where \(\mathcal{C}_K = c_h^{(K)} - c_l\). With \(\delta = 1\), it is easy to check that \(w_1^*(P, I, p) \leq \frac{w + P}{2}\) for all \(0 < p < 1\),\(^{17}\) which amounts to that the individual at time 0 always allocate \(w_1 \leq \frac{w + P}{2}\) for the first period. This yields an upper bound for the reward

\[
\mathcal{V}_K = \left( p_l - p_h^{(K)} \right) \left[ u(w - w_1) - u(w - w_1 - L + I) \right]
\]

\[
\leq \left( p_l - p_h^{(K)} \right) \left[ u \left( \frac{w - P}{2} \right) - u \left( \frac{w - P}{2} - L + I \right) \right]
\]

\[
\leq \left( p_l - p_h^{(K)} \right) \left[ u \left( \frac{w}{2} \right) - u \left( \frac{w}{2} - L \right) \right].
\]

At time 1-\(K\), if the individual takes the precautionary activity with high effort immediately, her utility is

\[
\beta' \mathcal{V}_K - \mathcal{C}_K.
\]

If the individual delays the activity to the next subperiod, her utility becomes

\[
\beta' \mathcal{V}_{K+1} - \beta' \mathcal{C}_{K+1} \text{ or } \beta' \mathcal{V}_{K+1} - \beta' \mathcal{C}_{K+1} - \varepsilon_K,
\]

(A21)

corresponding to whether the individual is sophisticated or naive. Therefore, condition (A17)

\(^{17}\)This follows straightforwardly from the first-order condition for \(w_1^*(P, I, p)\): \(u'(w_1^* - P) = (1 - p)u'(w - w_1^*) + pu'(w - w_1^* - L + I) \geq u'(w - w_1^*).\)
implies that once the task of taking high effort is delayed to the last time $1-N$, naifs will eventually give up. Condition (A18) implies that naifs procrastinate repeatedly since (A20) is smaller than (A21) with $\varepsilon_K \equiv 0$. Condition (A19) implies that sophisticates will take high effort at the time point $1-m$ since breaking the promise has become so costly at this time that it overweighs the utility gain.

Q.E.D.

References


