Abstract: The risk benchmarks and underwriting cycle models presented in this paper can be used by insurance firms in their own risk analysis and Enterprise Risk Management (ERM) modeling. The findings are based on a recent research study of the U.S. property-casualty insurance industry, building upon thousands of hours of data gathering of statutory filings. In the first part we analyze the historical underwriting cycle, develop a regime-switching model for simulating future cycles, and show its superiority to an autoregressive approach. In the second part we compute benchmarks for pricing and reserving risks for different lines of business and segments of the industry (Large National, Super Regional and Small Regional). We also compute the historical correlation of the ultimate loss ratio between lines of business, as well as the correlation of the changes in the reserve estimate between lines of business.

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1 Introduction

U.S. insurers are required by regulators to file annual financial statements. We have compiled and cleaned a historical database of these filings, which presents opportunities for a vast scope of analysis. Our analysis of this database has produced pricing and reserving risk benchmarks and an underwriting cycle model, which are presented in this paper and can be used by insurers in their own enterprise risk management (ERM) modeling.

Our data sources are from SNL Financial, the National Association of Insurance Commissioners (NAIC), and A.M. Best. Specifically, at the industry level, we compiled the data from Best’s Aggregates & Averages (1976-2010). At the individual company and line of business level, we have compiled and cleaned the data for:

- Gross and net paid, case incurred and IBNR loss triangles reported\(^1\) as of 1996 to 2009, for accident years 1987 to 2009.
- Gross and net premium triangles reported as of 1996 to 2009, for accident years 1987 to 2009.

In this paper, we have:

- Analyzed the historical underwriting cycle and modeled the future underwriting cycle (section 2).
- Analyzed the volatility of ultimate loss ratios by line of business and firm type (section 3.2).
- Studied the volatility of changes to reserves estimates, by analyzing how the ultimate loss changes from its estimation at 12 months of development from the year of accident, to 120 months development (section 3.3).
- Estimated the correlation between lines of business of the ultimate loss ratio and reserve development (section 3.4).

This paper differs from previous work in that:

- The underwriting cycle model in this paper, being a nonlinear regime-switching model, is fundamentally different from models developed in previous research papers, which are generally linear auto-regressive models.
- The data used in this paper is at the individual company group level, which is a fine level of granularity. While some previous research papers address firm-level behavior, typically only simple edits are used to eliminate suspect data. By contrast, we have made extensive efforts to identify and correct erroneous data.

The findings presented in this paper are an example of the possible analysis that can be done, now that we have compiled this database. For example, an evaluation of pricing and reserving risk benchmarks can be made for a more targeted group of companies, to provide more relevant benchmarks for any one insurer.

\(^1\) Report year in this context is the “as of” year of the annual statement.
2 The Underwriting Cycle

2.1 History of the Underwriting Cycle
Over the past few decades, the U.S. Property & Casualty insurance industry has experienced periodic changes in profitability known as the underwriting cycle. As can be seen in Figure 2.1, the all-lines ratio of calendar-year losses to premiums, gross of reinsurance, appears as a wavy pattern, with values ranging from 65% to almost 90%.

Figure 2.1: Total P&C Calendar Year Gross Loss Ratio

2.2 Capacity Constraint Theory
There are many theories about the causes and mechanics of the cycle. One of the more popular is the “capacity constraint” theory. This focuses on the dynamic relationship between pricing and surplus. The capacity constraint theory is illustrated by Figure 2.2, which distinguishes between inadequate and adequate levels of surplus (horizontal axis) and inadequate and adequate pricing (vertical axis).

Time lags in reporting and in the emergence of losses interfere with the ability of firms to anticipate changes or make quick adjustments. The result is the cycle.

---

2 In this paper, when we refer to “loss” we mean loss and defense cost and containment expense (DCC), more commonly known as allocated loss and adjustment expense (ALAE).
Figure 2.3 shows the historical pricing versus surplus cycles. Total Premium Share (TPS) is our proxy for pricing. It is the ratio of net written premium to private sector GDP (PSGDP), which we use as an exposure measure. The ratio of policyholder surplus to private sector GDP is used as our surplus measure.

Figure 2.3 shows two underwriting cycles that conform to the circular pattern predicted by the capacity constraint theory: one from 1970 to 1983, and another from 1994 to 2008. The two underwriting cycles are separated by a period of extraordinary losses and a surge in premiums from 1986 to 1992 known as the “liability crisis”. The 2010 figures are projected from partial-year data.
We can view Total Premium Share (TPS) as the key driver of the underwriting cycle. The TPS can be viewed as a proxy for a pricing index, as, all else being equal (market penetration, terms and conditions, etc.) an increase in the TPS should be matched by a corresponding decrease in the loss ratio. Of course, all else is not equal. Other factors mediate between the all-industry all-lines TPS and the LOB-specific underwriting experience of a particular firm; this will be the subject of a future paper.

2.3 The Role of the Underwriting Cycle in Enterprise Risk Management

Currently, the underwriting cycle is not a feature of most insurers’ enterprise risk management models. Distributions used show no autocorrelation – an upward movement in loss ratios or reserve estimates in one year has no bearing on whether there is an upward movement in the following year.

However, the existence of the underwriting cycle is undeniable. Figure 2.1 shows that in real life, a series of deteriorating years is reasonably likely, and, for many insurers, this is the scenario of greatest concern. Conversely, a series of deteriorating years is highly unlikely in a model with no underwriting cycle - this model will, perhaps materially, underestimate risk.

To remedy this, we propose below a model of the underwriting cycle which produces simulated outcomes that can be fed into an insurer’s enterprise risk management model.
2.4 Literature Review on Underwriting Cycle Models

There are many research papers devoted to theories and models of the underwriting cycle. Built upon previous work of Major (2007), we classify some papers into the following themes.

**Institutional factors:** Pricing involves forecasting based on historical results, resulting in the price estimate lagging the true loss cost and creating the underwriting cycle. Papers exemplifying this explanation include Venezian (1985), Cummins & Outreville (1987), Lamm-Tenant & Weiss (1997), and Chen et al. (1999). Venezian’s paper is noteworthy as an early explanation, often referred to as the “actuaries are dumb” theory because it assumes naïve regression-style extrapolation in ratemaking. Cummins & Outreville, however, show that reporting and regulatory delays could cause the second-order auto-regressive pattern Venezian observed even under the assumption that actuaries behave as rationally as possible. Recently, Clark (2010) demonstrated that in the presence of estimation errors, applying theoretically correct actuarial techniques naturally leads to cyclical behaviors.

**Competition:** Not all competitors have the same view of the future, with a “winner’s curse” (Thaler 1992) phenomenon pushing the group towards lower rates, even if all participants are behaving rationally (and they may not be). Examples of this reasoning include Feldblum (1990), Harrington & Danzon (1994), Harrington (2004), Fitzpatrick (2004), Baker (2005) and Alkemper and Mango (2005). Feldblum provides a detailed explanation of competitive mechanisms and firms alternating between underwriting strategies of aggressive growth and price maintenance.

**Supply and demand, capacity constraints, and shocks:** Since insurance needs capital to support it, any shock that reduces capital, such as a natural catastrophe, will reduce capacity and therefore raise prices as supply becomes constricted. Declining profits may be exacerbated by anti-selection as more favorable business exits first. Capital market frictions (costly external capital) mean that capital cannot be replaced quickly (another source of delay). Papers include Winter (1991, 1994), Gron (1990, 1994), Niehaus & Terry (1993), and Cummins & Danzon (1997).

**Economic linkages:** Profitability for an insurer is linked to investment income, and cost of capital is linked to the wider economy. Expected losses in some lines of business are affected by inflation, GNP growth, or unemployment. Therefore, cycles in the economy result in cycles in insurance. Examples include Wilson (1981), Doherty & Kang (1988), Haley (1993), Grace & Hotchkiss (1995), and Madsen et al. (2005).

**All of the above:** Schnieper (2005) constructs a model which incorporates all above types of theories. Fung et al. (1998) test the predominant theories for underwriting cycles by sophisticated statistical methods and find that no single theory can explain underwriting cycles completely.

The model presented in this paper, while motivated by the above theories, is not focused on testing any particular theory. It is aimed at capturing the asymmetrical features of the downward versus upward cycle paths.
2.5 Modeling the Underwriting Cycle

2.5.1 Required Properties of the Underwriting Cycle model

Our simulation model for the underwriting cycle starts with the capacity constraint theory insight into pricing behavior. It is strictly empirical, driven by the data, but its structure is informed by the mechanics of the insurance industry.

The key to the state of the cycle is the Total Premium Share (TPS), the ratio of net written premiums to private sector GDP. A time series of Total Premium Share is shown in Figure 2.4. We define two phases or regimes:

- **Softening (DOWN regime):** a year in which the Total Premium Share is lower than in the previous year
- **Hardening (UP regime):** a year in which the Total Premium Share is higher than in the previous year

![Figure 2.4: Total Premium Share (1967-2009)](image)

We list below four requirements for the behavior of the underwriting cycle model:

**Some degree of auto-correlation**

We observe from Figure 2.4 that:

- A softening (DOWN) year generally tends to be followed by another softening year.
- A hardening (UP) year tends to be followed by another hardening year.

Therefore, we require some degree of auto-correlation of the TPS from one calendar year to the next.

**Switching of phases is a function of the TPS**
We observe from Figure 2.4 that:

- A softening year is more likely to be followed by hardening when Total Premium Share is low.
- A hardening year is more likely to be followed by softening when the Total Premium Share is high.

Therefore, we require the phase — i.e., the direction of change of TPS — to be a function of TPS.

**Statistical behavior is different in the two phases**

We observe that:

- It takes more years for prices to go down than to go up. Normally, it only takes 2-3 years for prices to go up, but for a downward move, the period is much longer.
- A positive difference (indicating prices going up) tends to be larger than a negative difference.
- The volatilities are different in the hardening versus softening phases.

The first two observations above can be easily confirmed by the following stem plot (Figure 2.5) for the first difference of log(NWP/PSGDP)$^3$ data:

---

$^3$ \text{log} indicates natural logarithm in this paper.
The third observation, that volatilities are different in hardening versus softening phases can be validated by a statistical test, but before doing that, we introduce more formal notation. For any given year \( t \):

\[
Y_t = \log(\text{Total Premium Share}) = \log\left(\frac{NW/PSGDP}{Y_{t-1}}\right);
\]

\( Y_t - Y_{t-1} \) is called a backward difference.

\( Y_{t+1} - Y_t \) is called a forward difference.

The sign of the backward difference establishes the regime for the forward difference. That is:

- If at time \( t \), \( Y_t - Y_{t-1} \geq 0 \), i.e., the backward difference was positive (or zero), we say that \( Y_t \) is now in an **UP regime**.
- Similarly, if \( Y_t - Y_{t-1} < 0 \), i.e., the backward difference was negative, we say that \( Y_t \) is in a **DOWN regime**.

Scatter plots of the forward differences for those two regimes are shown in Figure 2.6. That is, the forward difference in those instances where the backward difference was positive is plotted in the UP regime graph, and the forward difference in those instances where the backward difference was negative or zero is plotted in the DOWN regime graph:

![Figure 2.6: UP and DOWN Regime Scatter Plots](image)

It can be seen that the forward difference in an UP regime (i.e., when the backward difference was positive or zero) is also likely to be positive or zero. However, we have some possibility of a downward move (i.e., switching to a DOWN regime). Similarly, the forward difference in a DOWN regime is also likely to be negative, but there is still some possibility of an upward move (i.e., switching to an UP regime).
After defining the two regimes, we determined whether those two groups of forward differences have different variance. A two-sample $F$-test for equal variances was performed. At a 7% significant level, the UP regime has a significantly larger variance than the DOWN regime, confirming our observation.

However, if we remove an apparent outlier in the DOWN regime (the triangle in the upper left of Figure 2.6’s DOWN chart, representing the 1984-85 transition, the start of the liability crisis hardening), we then see a highly significant (0.1% level) variance difference between these two groups.

Because of these statistical properties, we believe that separate treatment of UP and DOWN regimes is necessary. This rules out the use of a simple auto-regressive (AR) model.

**It should explain itself; no exogenous variables should be included**

If this requirement is violated, meaning we include exogenous variables, then we have to make efforts to simulate those exogenous variables. We believe that private sector GDP is an appropriate proxy for exposure at the industry level. This is an exogenous variable and is available from government sources for calculating historical values of TPS. Note, however, that it is not needed to simulate future values.

2.5.2 Proposed Model: Regime-Switching Model

*Model Structure*

We propose the following model structure that satisfies all four requirements listed above. Future Total Premium Share (TPS) is simulated using a regime-switching Markov model based on its historical values and the current regime. Specifically, given a starting point $Y_0$ and a starting regime, we can simulate future $Y_t$ using:

- If $Y_t - Y_{t-1} > 0$, i.e., $Y_t$ is in an **UP regime**, the forward difference is given by:
  \[ Y_{t+1} - Y_t = f_{UP}(Y_t) + \varepsilon_{UP,t} \]

- If $Y_t - Y_{t-1} < 0$, i.e., $Y_t$ is in a **DOWN regime**, the forward difference is given by:
  \[ Y_{t+1} - Y_t = f_{DOWN}(Y_t) + \varepsilon_{DOWN,t} \]

It remains to find suitable functions $f_{UP}$ and $f_{DOWN}$, and distributions for the residual terms $\varepsilon_{UP}$ and $\varepsilon_{DOWN}$.

*Fitting the Model*

For the UP regime, the data appears to have a linear pattern, so a linear regression for $f_{UP}$ was undertaken (Figure 2.7).
The linear equation takes the following form:

\[ Y_{t+1} - Y_t = -0.4891 - 0.1597 \cdot Y_t + \varepsilon_{UP,t} \]

The variance of residual terms is 0.0032. Due to the small sample size, the histogram (not shown) of residuals \( \varepsilon_t \) does not show a bell curve shape. However, a Lilliefors test indicates that we could not reject the null hypothesis that the residuals are normally distributed\(^4\). A Chi-square goodness-of-fit test gave us the same result: we could not reject the null hypothesis. For both tests, the significance level is set at 5%. Therefore, we will assume that \( \varepsilon_{UP,t} \sim N(0, 0.0032) \)

For the DOWN regime, we identify a non-linear pattern.

---

\(^4\) Also, this model passed the Breusch–Pagan (B-P) test, thus we do not need to worry about heteroscedasticity.
We used a “hockey stick” model to fit the empirical data. Specifically, we fit the following model:

\[ y_{t+1} - y_t = a + b \cdot \min(y_t, c) + \varepsilon_{\text{DOWN}, t} \]

where \(a\), \(b\) and \(c\) are parameters to be estimated. This is a non-linear model, which could be estimated by either Maximum Likelihood Estimation (MLE) or Generalized Least Squares (GLS). However, these methods turned out to be unstable on this data. Therefore, we used a consistent two-stage algorithm to estimate the \(c\) parameter first, and the results were confirmed by both MLE and GLS.

The “hockey stick” pattern was also confirmed by locally weighted scatter plot smoothing (LOESS), which used bootstrapping and cross-validation to determine an optimal span value (Figure 2.9). \(^5\)

\(^5\) For technically inclined readers, LOESS uses a tri-cube weight function: \(w(x) = (1 - x^3)^3\), where \(x\) is the distance scaled to the span value.
Between LOESS and Hockey Stick, we chose the latter, because:

1. LOESS gave too much weight to neighbor points, leading to insufficient smoothing. For example, the convex pattern between -3.2 and -3.1 was judged not more credible than a horizontal line (as the Hockey Stick model indicates.) In other words, LOESS may cause an over-fitting problem.

2. Due to the small sample size, the training set for cross-validation is also small, causing the optimal span value for LOESS to be volatile.

We believe that the Hockey Stick model represents the pattern well. It is also computationally more efficient than LOESS.

The equation we estimated is:

\[ Y_{t+1} - Y_t = -2.4358 - 0.7266 \cdot \min(Y_t, -3.3129) + \epsilon_{DOWN,t} \]

The variance of residual terms is 0.0012. Again, at a 5% significance level, we cannot reject the normal distribution hypothesis for residuals, using both the Lilliefors test and a Chi-square goodness-of-fit test.\(^6\)

Therefore, we will assume that \( \epsilon_{DOWN,t} \approx N(0,0.0012) \).

\(^6\) The Hockey Stick model also passed the B-P test for heteroscedasticity.
Simulation Results

The preceding fully specifies the procedure to simulate $Y_t = \log(\frac{NWP}{PSGDP})$ and therefore also Total Premium Share = NWP / PSGDP = $\exp(Y_t)$. Figure 2.10 gives four examples of paths generated from the simulation model.

Figure 2.10: Example Simulation Paths of Total Premium Share

For a 5,000 iteration simulation, Figure 2.11 gives a summarized box plot for each accident year.
On each box, the central red mark represents the median, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points.

The TPS is 3.43% in calendar year 2009. To estimate 2010 data, we use information from the first three quarters in this year, rather than a regime-switching model. Starting from year 2011, the simulation method is implemented. The 2011 results are distributed approximately symmetric around the 2009 level, and the variation is narrow. For calendar year 2012 and beyond, the right tail is growing, indicating a higher possibility of price increments.

Figure 2.12 shows comparable simulation results from an AR(2) model fit to the TPS data. (A lag of 2 was selected by the Akaike Information Criterion as the most suitable up to 5 lags.) The future median converges to 0.0332, which is lower than both 1984 and 2000 TPS (shown in Figure 2.4 to be the bottoms of two soft markets.) This strange result is mainly due to the linear/symmetric nature of the AR model.
3 Pricing Risk and Reserving Risk Benchmarks

3.1 Data

We have compiled data from the annual statement Schedule P for each U.S. insurer, by line of business, both gross and net of reinsurance, for accident years 1987 to 2009. This includes paid, incurred and IBNR loss and expense triangles as well as earned premium.

Moreover, we have tested the data for consistency from one annual statement year to the next. The data must be adjusted for mergers and acquisitions activity, and for other changes that result in a restatement of historical data.

In order to clearly define pricing and reserving risk benchmarks, we need to make the following clarifications:

**Gross or net of reinsurance:** We note that companies may have different results before and after applying reinsurance. We calculate risk benchmarks using both gross and net premium and loss triangles. Note that while the net loss triangles have 23 accident years (1987-2009), we only have 14 accident years of gross loss triangles data (1996-2009).

**Unit of observation:** We need to specify the *unit of observation.* Computed volatility depends on the data sample. A small premium size tends to generate higher statistical volatility of loss ratios. Consolidating premium and loss data across companies will reduce the statistical volatility of loss ratios.
We have defined the unit of observation as either consolidated for the whole industry, consolidated for all companies in a segment and for each company group.

To determine the company group level, we used SNL insurance company groups (and subgroups on some occasions). A company group may have many operating companies. We only keep company groups whose minimum entry in the premium triangle is at least $1 million. This leaves us with 730 company groups.

To determine the segment level, we divided the 730 company groups into seven segments, listed in Table 3.1.
In this paper, we estimate risk benchmarks for the following segments: 1) Large National, 2) Super Regional, and 3) Small Regional.

Please keep in mind the heterogeneity of individual firms’ experience. While individual firms tend to correlate with the industry-wide experience, an individual firm’s volatility is also affected by its own corporate strategy, underwriting practice and a host of other firm-specific factors. While it would be interesting to analyze different growth strategies and firm characteristics, in this paper we simply classify firms according to the markets in which they operate (Large National, Super Regional, and Small Regional).

**Lines of Business:** We study benchmarks of pricing risks and reserving development risks for eight major lines of business (LOB).

1) Private Passenger Auto Liability  
2) Commercial Auto Liability  
3) Workers Compensation  
4) Other Liability Occurrence and Claims-Made combined (i.e., general liability),  
5) Product Liability Occurrence  
6) Medical Professional Liability Occurrence and Claims-Made combined  
7) Commercial Multiple Peril  
8) Homeowners

Product Liability Occurrence and Medical Malpractice Liability are not separated into three segments due to the small number of companies in these lines of business.

The appendix provides more detail on our data and segmentation methodology.

### 3.2 Pricing Risk Benchmarks

#### 3.2.1 Coefficient of Variation of the Gross Loss Ratio by Line of Business and Segment

Figure 3.1 and its accompanying Table 3.2 show the coefficient of variation (standard deviation / mean) of the gross loss ratio by segment and line of business.
Table 3.2: Coefficient of Variation of the Aggregate Gross Loss Ratio across AY 1996-2004, as shown in Figure 3.1

<table>
<thead>
<tr>
<th>Product Liability Occurrence</th>
<th>Total</th>
<th>Large National</th>
<th>Super Regional</th>
<th>Small Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other Liability</td>
<td>0.33</td>
<td>0.31</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td>Medical Professional Liability</td>
<td>0.25</td>
<td>0.22</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td>Commercial Multiple Peril</td>
<td>0.21</td>
<td>0.21</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>Homeowners</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers’ Compensation</td>
<td>0.21</td>
<td>0.16</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Commercial Auto Liability</td>
<td>0.15</td>
<td>0.11</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Private Passenger Auto</td>
<td>0.07</td>
<td>0.07</td>
<td>0.06</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.1 shows that

1. Aggregated by segment and line of business, the ultimate reported gross loss ratios vary across accident years the most for Product Liability and the least for Private Passenger Auto.
2. The Large National segment shows a higher coefficient of variation than do the Super Regional and Small Regional segments. This pattern holds for almost every line of business, with the exception being Private Passenger Auto Liability. This suggests important differences in pricing behaviors between firms in different segments. A similar finding was reported in Wang and Faber (2006) which was based on a limited sample analysis.
3.2.2 Coefficient of Variation of the Net Loss Ratio by Line of Business and Segment

Figure 3.2 shows the coefficient of variation of the net loss ratio by segment and line of business. Observations can be made on the net loss ratios that are similar to those on the gross loss ratios.

![Figure 3.2: Coefficient of Variation of the Aggregate Net Loss Ratio across AY 1987-2004 (by Segments and Lines of Business)](image)

Table 3.3: Coefficient of Variation of the Aggregate Net Loss Ratio across AY 1996-2004, as shown in Figure 3.2

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Large National</th>
<th>Super Regional</th>
<th>Small Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product Liability Occurrence</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Liability</td>
<td>0.25</td>
<td>0.13</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Medical Professional Liability</td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial Multiple Peril</td>
<td>0.18</td>
<td>0.19</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Homeowners</td>
<td>0.22</td>
<td>0.12</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Workers’ Compensation</td>
<td>0.18</td>
<td>0.16</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>Commercial Auto Liability</td>
<td>0.13</td>
<td>0.10</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Private Passenger Auto</td>
<td>0.08</td>
<td>0.07</td>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>

3.2.3 Variability of the Ultimate Loss Ratio for Large Firms versus Small Firms

Previously, we saw that the aggregate loss ratio variation is lowest for Private Passenger Auto in the Small Regional segment and highest for Product Liability Occurrence. Here, we explore that variation across individual firms, and, in particular, ask whether larger firms experience less variation than smaller firms (as might be expected).
In the following charts, each dot represents one firm. The horizontal axis represents the average, across accident years, of the net earned premium for the firm in the specified LOB. A logarithmic scale is used because of the large variation across companies. The vertical axis measures the standard deviation of the ultimate loss ratio for the firm in the specified LOB, gross or net, across accident years.

![Figure 3.3: Standard Deviation of Net ULR vs. Premium for Private Passenger Auto](image)

\[ y = -0.012\ln(x) + 0.2247 \]
\[ R^2 = 0.2621 \]

With 135 companies writing Private Passenger Auto business (Figure 3.3), we can see that larger firms do tend to have lower volatility than small firms, as implied by the law of large numbers.

Such a significant negative relationship can also be found in Commercial Auto Liability (shown in Figure 3.4), with a sample of 99 companies.
For Other Liability, however, we get a counterintuitive but statistically significant result: among 99 companies, larger firms are more volatile, as shown in Figure 3.5. This suggests systematic effects of firm size on pricing behavior.

Other lines of business did not show significant relationships in either direction.
3.3 Reserving Risk Benchmarks

3.3.1 Reserving Risk Benchmarks by Line of Business

The following two charts show the historical reserve development, expressed in terms of revisions of reported accident year loss ratios. The horizontal axis represents the accident year. The vertical axis measures the difference between latest report (at 120 months of development or less)\(^7\) and first report (at 12 months of development) of the industry aggregate loss ratio for a specified line of business on both a net and gross basis.

We call the latest reported loss ratio the Ultimate Loss Ratio, and the first reported loss ratio the Initial Loss Ratio. The reserve risk benchmark is defined as the maximum difference between the Ultimate Loss ratio and the Initial Loss ratio across all accident years. For each graph, we label where this maximum occurs.

Figure 3.6: Accident Year Ultimate Loss Ratio minus Initial Loss Ratio for Commercial Auto Liability

The reserving risk for Commercial Auto Liability on a gross basis is 15%, the maximum of \{Ultimate Loss Ratio - Initial Loss Ratio\} across accident years 1989-2004. The reserve risk for Commercial Auto Liability on a net basis is 19%, the maximum of \{Ultimate Loss Ratio - Initial Loss Ratio\} across accident years 1980-2004. It is noted that the reserving risk for gross versus net are based on data from different time spans and thus are not directly comparable.

In Figure 3.7, the reserving risk for Other Liability is shown.

---

\(^7\) Schedule P reports the estimate of ultimate loss for an accident year, at 12 month evaluations up to 120 months before the accident year falls off the Schedule. The most recent Schedule P available is as of 2009. Therefore, the latest report for, say, accident year 2004 will be at 72 months (2009), not 120 months of development.
A summary of the reserving risk for all lines of business is in Table 3.4.

Table 3.4: Reserving Risk by Line of Business: Maximum of Gross and Net ULR – ILR

<table>
<thead>
<tr>
<th></th>
<th>Commercial Auto Liability</th>
<th>Commercial Multi Peril</th>
<th>Homeowners</th>
<th>Medical Professional Liability</th>
<th>Other Liability</th>
<th>Private Passenger Auto</th>
<th>Workers’ Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gross</strong></td>
<td>15%</td>
<td>12%</td>
<td>4%</td>
<td>24%</td>
<td>43%</td>
<td>1%</td>
<td>20%</td>
</tr>
<tr>
<td><strong>Net</strong></td>
<td>19%</td>
<td>16%</td>
<td>5%</td>
<td>51%</td>
<td>65%</td>
<td>4%</td>
<td>20%</td>
</tr>
</tbody>
</table>

3.3.2 Reserving Risk Benchmark by Line of Business and Segment

The preceding section measured reserving risk at the line of business level. To investigate whether reserving risk differs across different segments, we study aggregate segment level data (with net loss data for accident-years 1987-2004, and gross loss data for accident-years 1996-2004).

Using consolidated gross loss ratio data of all companies within a segment, we get the following segment-level reserve risk benchmarks. Due to high variation of loss ratios, we measure a relative, rather than absolute, change in loss ratio: the Ultimate Loss Ratio divided by the Initial Loss Ratio less 1 (ULR / ILR – 1).
Using consolidated net loss ratio data of all companies within a segment, we get the following reserving risk benchmarks.

### Table 3.6: Reserving Risk by Line of Business and Segment: Maximum of Net ULR/ILR-1

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Large National</th>
<th>Super Regional</th>
<th>Small Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product Liability Occurrence</strong></td>
<td>74%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Other Liability</strong></td>
<td></td>
<td>47%</td>
<td>(3)%</td>
<td>96%</td>
</tr>
<tr>
<td><strong>Workers’ Compensation</strong></td>
<td>29%</td>
<td>13%</td>
<td>14%</td>
<td></td>
</tr>
<tr>
<td><strong>Commercial Auto Liability</strong></td>
<td>18%</td>
<td>6%</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td><strong>Medical Professional Liability</strong></td>
<td>17%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Commercial Multi Peril</strong></td>
<td>14%</td>
<td>7%</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td><strong>Homeowners</strong></td>
<td>7%</td>
<td>2%</td>
<td>(1)%</td>
<td></td>
</tr>
<tr>
<td><strong>Private Passenger Auto Liability</strong></td>
<td>2%</td>
<td>(1)%</td>
<td>(1)%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Observe that Large Nationals (on a consolidated basis) again tend to show higher reserve risk benchmarks than Super Regional and Small Regional companies. This, again, is probably due to differences in behavior.

### 3.4 Correlation Benchmarks

#### 3.4.1 Correlation of Accident Year Ultimate Loss Ratios Between Lines of Business

Using A.M. Best data for accident year 1980-2006, we calculated correlations of ultimate loss ratios aggregated for the industry, across accident years, between lines of business, using Spearman’s rank correlation. For example, if, for two lines of business, the highest loss ratio\(^8\) falls in the same accident year, and the second highest loss ratio for both lines also falls in the same accident year, and so on, then we will calculate a 100% correlation between the two lines.

For both Table 3.7 and Table 3.8, correlations over 38% are statistically significant at the p=5% level.

\(^{8}\) That is, the highest loss ratio for that line of business across all accident years.
Table 3.7: Correlation of Accident Year Ultimate Loss Ratios between Lines of Business

<table>
<thead>
<tr>
<th>LOB</th>
<th>Total</th>
<th>Other Liability</th>
<th>Commercial Multi Peril</th>
<th>Homeowners</th>
<th>Medical Professional Liability</th>
<th>Commercial Auto Liability</th>
<th>Private Passenger Auto Liability</th>
<th>Workers' Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>100%</td>
<td>82%</td>
<td>85%</td>
<td>50%</td>
<td>62%</td>
<td>79%</td>
<td>47%</td>
<td>56%</td>
</tr>
<tr>
<td>Other Liability</td>
<td>82%</td>
<td>100%</td>
<td>83%</td>
<td>24%</td>
<td>88%</td>
<td>81%</td>
<td>20%</td>
<td>40%</td>
</tr>
<tr>
<td>Commercial Multi Peril</td>
<td>85%</td>
<td>83%</td>
<td>100%</td>
<td>46%</td>
<td>72%</td>
<td>79%</td>
<td>13%</td>
<td>27%</td>
</tr>
<tr>
<td>Homeowners</td>
<td>50%</td>
<td>24%</td>
<td>46%</td>
<td>100%</td>
<td>13%</td>
<td>12%</td>
<td>8%</td>
<td>-5%</td>
</tr>
<tr>
<td>Medical Professional Liability</td>
<td>62%</td>
<td>88%</td>
<td>72%</td>
<td>13%</td>
<td>100%</td>
<td>73%</td>
<td>-8%</td>
<td>8%</td>
</tr>
<tr>
<td>Commercial Auto Liability</td>
<td>79%</td>
<td>81%</td>
<td>79%</td>
<td>12%</td>
<td>73%</td>
<td>100%</td>
<td>34%</td>
<td>52%</td>
</tr>
<tr>
<td>Private Passenger Auto Liability</td>
<td>47%</td>
<td>20%</td>
<td>13%</td>
<td>8%</td>
<td>-8%</td>
<td>34%</td>
<td>100%</td>
<td>75%</td>
</tr>
<tr>
<td>Workers’ Compensation</td>
<td>56%</td>
<td>40%</td>
<td>27%</td>
<td>-5%</td>
<td>8%</td>
<td>52%</td>
<td>75%</td>
<td>100%</td>
</tr>
</tbody>
</table>

3.4.2 Correlation Matrix of Accident Year Reserve Development across Lines of Business

The correlation of reserve development is done by calculating the correlation of the Ultimate Loss ratio divided by the Initial Loss ratio less 1 (ULR / ILR - 1). This also uses Spearman’s rank correlation.

Table 3.8: Correlation of Accident Year Reserve Development between Lines of Business

<table>
<thead>
<tr>
<th>LOB</th>
<th>Total</th>
<th>Other Liability</th>
<th>Commercial Multi Peril</th>
<th>Homeowners</th>
<th>Medical Professional Liability</th>
<th>Commercial Auto Liability</th>
<th>Private Passenger Auto Liability</th>
<th>Workers’ Compensation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>100%</td>
<td>86%</td>
<td>81%</td>
<td>57%</td>
<td>76%</td>
<td>91%</td>
<td>85%</td>
<td>82%</td>
</tr>
<tr>
<td>Other Liability</td>
<td>86%</td>
<td>100%</td>
<td>74%</td>
<td>49%</td>
<td>84%</td>
<td>75%</td>
<td>62%</td>
<td>55%</td>
</tr>
<tr>
<td>Commercial Multi Peril</td>
<td>81%</td>
<td>74%</td>
<td>100%</td>
<td>51%</td>
<td>60%</td>
<td>80%</td>
<td>56%</td>
<td>63%</td>
</tr>
<tr>
<td>Homeowners</td>
<td>57%</td>
<td>49%</td>
<td>51%</td>
<td>100%</td>
<td>38%</td>
<td>51%</td>
<td>51%</td>
<td>49%</td>
</tr>
<tr>
<td>Medical Professional Liability</td>
<td>76%</td>
<td>84%</td>
<td>60%</td>
<td>38%</td>
<td>100%</td>
<td>81%</td>
<td>65%</td>
<td>38%</td>
</tr>
<tr>
<td>Commercial Auto Liability</td>
<td>91%</td>
<td>75%</td>
<td>80%</td>
<td>51%</td>
<td>81%</td>
<td>100%</td>
<td>82%</td>
<td>72%</td>
</tr>
<tr>
<td>Private Passenger Auto Liability</td>
<td>85%</td>
<td>62%</td>
<td>56%</td>
<td>51%</td>
<td>65%</td>
<td>82%</td>
<td>100%</td>
<td>80%</td>
</tr>
<tr>
<td>Workers’ Compensation</td>
<td>82%</td>
<td>55%</td>
<td>63%</td>
<td>49%</td>
<td>38%</td>
<td>72%</td>
<td>80%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The reserve development correlation matrix shows higher correlations than are generally seen when estimating the correlation between incremental losses in the triangle, which is another method to obtain correlations. However, the incremental loss method is too granular – the correlation between incremental losses is not the correlation of the whole reserve, as each individual incremental loss is not
independent of the next. Instead, there exists autocorrelation – that is, there is a reserving cycle that is a common thread through all lines of business.

4 Conclusions
In this paper, we have modeled the underwriting cycle using the Total Premium Share (net written premium over private sector GDP) as a proxy for the pricing index. By observing the historical behavior of the TPS, we determined that the downward and upward regimes of the cycle behave in different ways, and so a regime-switching model was created instead of the usual autoregressive model. Using our extensive database of 730 company groups, we also estimated benchmark pricing and reserving risks by line of business and by industry segment (Large National, Super Regional and Small Regional), as well as the correlation between lines of business at an industry-wide level.

These results can be used as a guide in selecting risk parameters for enterprise risk models as well as augmenting those models to include explicit underwriting cycle effects.
References


Appendix

Data and Segmentation
Our main source of data is Schedule P triangles (available from SNL, NAIC, and A.M. Best).

In determining the building blocks for calculating risk benchmarks, we agreed to use SNL insurance company groups (and subgroups on some occasions). We compiled data for eight major lines of business (LOB):

1) Private Passenger Auto Liability  
2) Commercial Auto Liability  
3) Workers Compensation  
4) Other Liability Occurrence and Claims-Made combined  
5) Product Liability Occurrence  
6) Medical Professional Liability Occurrence and Claims-Made combined  
7) Commercial Multiple Peril  
8) Homeowners

We compiled raw Schedule P triangle data by lines of business:

1) Gross Paid, Case Incurred and INBR loss triangles reported as of 1996 to 2009  
2) Net Paid, Case and IBNR loss triangles from 1987 to 2009 (reported as of 1996 to 2009)  
3) Gross and Net Premium triangles from 1987 to 2009 (reported as of 1996 to 2009)

SNL company group data already reflects past mergers and acquisitions. We performed a diagnosis of data consistency and found that despite SNL efforts, numerous inconsistencies exist due to past mergers and acquisitions. This is especially pronounced for large company groups. We examined company group data along with data for their member companies. In that process, we identified several inconsistencies:

1) Restatement of historical data (under new regulations a previous transaction does not meet the test of risk transfer and must be treated as deposit accounting).  
2) Company reported a number with a higher (or lower) value in one annual statement year but reduced (or increased) the same amount in the next year.  
3) Inter-company reinsurance.  
4) Missing companies from the company group.

We examined the merger and acquisition history of large company groups and identified the characteristics of member companies of large company groups. Some of the member companies are reinsurers and were taken out from the company group data.

We focused our data cleaning efforts on diagnosis of data consistency and reasonableness of loss development factors. We applied diagnosis and repairing algorithms to clean P&C insurance companies’ Gross (Paid and Case Incurred) loss triangles.

We further grouped companies into seven segments:
<table>
<thead>
<tr>
<th>Segment</th>
<th>Number of Company Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large National</td>
<td>23</td>
</tr>
<tr>
<td>Super Regional</td>
<td>30</td>
</tr>
<tr>
<td>Small Regional</td>
<td>474</td>
</tr>
<tr>
<td>Specialty Writer</td>
<td>55</td>
</tr>
<tr>
<td>Reinsurer</td>
<td>19</td>
</tr>
<tr>
<td>Other</td>
<td>113</td>
</tr>
<tr>
<td>Florida Homeowner</td>
<td>16</td>
</tr>
</tbody>
</table>

Below are the criteria for the first three segments.

Large national: Company groups that satisfy both

1) 2009 Direct Written Premium is more than $1B in commercial lines, and
2) 2009 All Lines Direct Written Premium in any of 6 geographic regions\(^9\) of US is less than 50%  
   - USAA Insurance Group (SNL P&C Group) is an exception. Its 2009 All Lines Direct Written Premium is more than 10B, and 2009 Direct Written Premium in commercial lines is $720M. We grouped it in Large National.

Super Regional: Those companies not in Large National and with 2009 All Lines Direct Written Premium being more than $1B.

- Old Republic International (SNL P&C Group) is an exception. Its 2009 Direct Written Premium in commercial lines is $3.3B, but it is more of a conglomeration of large regional companies and specialty writers and likely operates more like a Super Regional or Specialty Writer than a Large National. We grouped it in Super Regional.
- Assurant Inc. (SNL P&C Group) is another exception. Its 2009 Direct Written Premium in commercial lines is $3B, but most of its business is in Special Property and A&H. We grouped it in Super Regional.

Small Regional: Those companies not in Large National, Super Regional, Specialty Writer, or Reinsurer.

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\(^9\) Mid Atlantic, midwest, northeast, southeast, southwest, west.